Unbiased simulation of stochastic differential equations

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Abstract

We propose an unbiased Monte-Carlo estimator for E[g(X_t1,,X_tn)], where X is a diffusion process defined by a multi-dimensional stochastic differential equation (SDE). The main idea is to start instead from a well-chosen simulatable SDE whose coefficients are updated at independent exponential times. Such a simulatable process can be viewed as a regime-switching SDE, or as a branching diffusion process with one single living particle at all times. In order to compensate for the change of the coefficients of the SDE, our main representation result relies on the automatic differentiation technique induced by Bismu-Elworthy-Li formula from Malliavin calculus, as exploited by Fournie et al.(1999) for the simulation of the Greeks in financial applications. In particular, this algorithm can be considered as a variation of the (infinite variance) estimator obtained in Bally and Kohatsu-Higa [Section 6.1](2014) as an application of the parametrix method.

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