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# Ninomiya-Victoir scheme: strong convergence, antithetic version and application to multilevel estimators

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## Abstract

In this paper, we are interested in the strong convergence properties of the Ninomiya-Victoir scheme which is known to exhibit weak convergence with order 2. We prove strong convergence with order 1/2. This study is aimed at analysing the use of this scheme either at each level or only at the finest level of a multilevel Monte Carlo estimator: indeed, the variance of a multilevel Monte Carlo estimator is related to the strong error between the two schemes used on the coarse and fine grids at each level. Recently, Giles and Szpruch proposed a scheme permitting to construct a multilevel Monte Carlo estimator achieving the optimal complexity  $O(-2)$  for the precision. In the same spirit, we propose a modified Ninomiya-Victoir scheme, which may be strongly coupled with order 1 to the Giles-Szpruch scheme at the finest level of a multilevel Monte Carlo estimator. Numerical experiments show that this choice improves the efficiency, since the order 2 of weak convergence of the Ninomiya-Victoir scheme permits to reduce the number of discretization levels.

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