An Augmented Lagrangian Method for Mean Field Type Control with Congestion

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Abstract

The theory of mean field type control aims at describing the behaviour of a large number of interacting agents using a common feedback. A type of problems that have recently raised a lot of interest concerns congestion effects. These problems model situations in which the cost of displacement of the agents increases in regions where the density is large (as, for instance, in crowd motion). We obtain existence and uniqueness of suitably defined weak solutions for a system of partial differential equations arising in this setting. The solutions are characterized as the optima of two optimal control problems in duality. To solve numerically this problem, a monotone finite difference scheme is proposed and shown to have a variational interpretation. Then an augmented Lagrangian is defined and an Alternating Direction Method of Multipliers is proposed for solving the latter variational problem. Two kinds of boundary conditions are considered: periodic conditions and boundary conditions associated to state constrained problems which allows one to model more realistic situations. Various test cases and numerical results are presented.

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