Multi-level and Multi-index Monte Carlo

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Abstract

We will first discuss adaptive strategies in the context of Multilevel Monte Carlo (MLMC) Methods for (i) Itô Stochastic Differential Equations, (ii) Stochastic Reaction Networks modeled by Pure Jump Markov Processes and (iii) Partial Differential Equations with random inputs. Here, the notion of adaptivity includes several aspects such as mesh refinements based on either a priori or a posteriori error estimates, the local choice of different time stepping methods and the selection of the total number of levels and the number of samples at different levels. Our Adaptive MLMC estimator uses a hierarchy of adaptively refined, non-uniform time discretizations, and, as such, it may be considered a generalization of the uniform discretization MLMC method introduced independently by M. Giles and S. Heinrich. In particular, we show that our adaptive MLMC algorithms are asymptotically accurate and have the correct complexity with an improved control of the multiplicative constant factor in the asymptotic analysis. We also developed techniques for estimation of parameters needed in our MLMC algorithms, such as the variance of the difference between consecutive approximations. These techniques take particular care of the deepest levels, where for efficiency reasons only a few realizations are available to produce essential estimates. Moreover, we show the asymptotic normality of the statistical error in the MLMC estimator, justifying in this way our error estimate that allows prescribing both the required accuracy and confidence level in the final result.

In the second part of this talk, we describe and analyze the Multi-Index Monte Carlo (MIMC) for computing statistics of the solution of a PDE with random data. MIMC is both a stochastic version of the combination technique introduced by Zenger, Griebel and collaborators and an extension of the Multilevel Monte Carlo (MLMC) method first described by Heinrich and Giles. Instead of using first-order differences as in MLMC, MIMC uses mixed differences to reduce the variance of the hierarchical differences dramatically. These mixed differences yield new and improved complexity results, which are natural generalizations of Giles's MLMC analysis, and which increase the domain of problem parameters for which we achieve the optimal convergence. We finally show the effectiveness of MIMC in some computational tests, including PDEs with random coefficients and Stochastic Particle Systems.

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