International conference on Monte-Carlo techniques

5-8 Jul 2016

Paris, France



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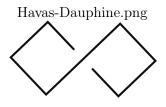
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Table of contents

Tuesday, July 5th

Plenary talks	8
Recent Developments in Randomized MLMC, Peter Glynn	8
Monte Carlo Techniques in Modern Stochastic Optimization for Big Data Machine Learning, Tong Zhang	8
Some applications of importance sampling to dependability analysis, Bruno Tuffin	9
Minisymposium: Unbiased simulation of SDEs	9
A stochastic parametrix method, Vlad Bally	9
Unbiased simulation of stochastic differential equations using parametrix expan- sions, Patrik Andersson	10
Unbiased simulation of stochastic differential equations, Xiaolu Tan	10
Minisymposium: Particle methods	11
Anytime Monte Carlo, Lawrence Murray	11
On the two-filter approximations of marginal smoothing distributions in general state space models, Sylvain Le Corff [et al.]	11
Variance estimation in the particle filter, Nick Whiteley	12
Minisymposium: Quasi Monte-Carlo	12

 $\mathbf{7}$

Minisymposium: Quasi Monte-Carlo

Hermite spaces and QMC methods in quantitative finance, Leobacher Gunther $% \mathcal{A}$.	12
Fast QMC matrix vector multiplication in option pricing, Josef Dick [et al.] $\ . \ .$	13
Sequential quasi-Monte Carlo, Nicolas Chopin [et al.]	13
Minisymposium: Model risk and uncertainty	13
Volatility derivatives and model-free implied leverage, Masaaki Fukasawa	14
Consistency and model uncertainty in affine interest rate models, Philipp Harms	14
Kriging of financial term-structures, Areski Cousin [et al.]	15
Wednesday, July 6th	16
Plenary talks	17
To be announced, Terry Lyons	17
Numerical solution of the master equation arising in large population stochastic control, Jean-François Chassagneux	17
Stochastic Kriging for Bermudan Option Pricing, Michael Ludkovski	18
Minisymposium: Simulation of BSDEs	18
Simulation of BSDEs with jumps by Wiener chaos expansion, Céline Labart [et al.]	18
Adaptive importance sampling in least-squares Monte Carlo algorithms for back- ward stochastic differential equations, Plamen Turkedjiev [et al.]	19
Numerical approximation of switching problems, Adrien Richou [et al.] $\ . \ . \ .$	19
Minisymposium: Simulation of stochastic graphs and applications	19
Community Detection with the Non-Backtracking Operator, Marc Lelarge $\ . \ . \ .$	20
Default Cascades in Inhomogeneous Financial Networks, Hamed Amini $[{\rm et\ al.}]$	20

Optimal Control for financial system with default contagion, Agnes Sulem [et al.]	21
Minisymposium: HPC and GPU	22
Resolution of a large number of small random symmetric linear systems in single precision arithmetic on GPUs, Lokmane Abbas Turki [et al.]	22
Stratified Nested Regression Monte-Carlo scheme with large scale paralleliza- tion, Emmanuel Gobet [et al.]	22
Toward a benchmark GPU platform to simulate XVA, Babacar Diallo	23
Minisymposium: Optimal trading	23
Probability of Fill Estimation Using Trading Signals, Aymen Jedidi	23
Optimal Execution with Statistical Learning, Sophie Laruelle	24
Quants at work: testing prototypes via Monte Carlo simulations, Mauricio Labadie	24
Poster session	24
A Study of the Application of Chaos to the Global Optimization., Tayeb Hamaizia	25
Hamiltonian Monte Carlo sampling for Wishart distributions with eigenvalue con- straints, Alexander Buchholz [et al.]	25
Heterogeneous Risk Preferences in Financial Markets, Tyler Abbot	26
Improved adaptive Multilevel Monte Carlo and applications to finance, Kaouther Hadji [et al.]	26
Parareal methods and applications in finance, Guillaume Sall	27
Thursday, July 7th	28

Plenary talks	29
Two new MLMC applications, Michael Giles	29

Nonlinear stochastic ordinary and partial differential equations: regularity prop- erties and numerical approximations, Arnulf Jentzen	29
Automatic Differentiation in Machine Learning: Survey and Revisionist History, Bara Pearlmutter	
Minisymposium: Efficient computations of sensitivities	30
Automatic Differentiation for Complex options and algorithms, Olivier Pironneau	30
Practical implementation of adjoint algorithmic differentiation (AAD), Sébastien Geeraert	31
AAD applications in Finance, Adil Reghai	31
Minisymposium: SDE approximation	32
Rate of convergence of the Euler scheme for SDE with discontinuous drift, Antoine Lejay	32
Trajectorial coupling between one-dimensional diffusions with linear diffusion co- efficient and their Euler scheme, Arnaud Gloter [et al.]	32
A Symmetrized Milstein scheme with strong rate of convergence for some CEV-like SDEs, Mireille Bossy	33
Minisymposium: Multilevel Monte Carlo	33
General multilevel adaptations for stochastic approximation algorithms, Steffen Dereich [et al.]	33
Multilevel Monte Carlo for McKean-Vlasov SDEs, Lukasz Szpruch	34
Ninomiya-Victoir scheme: strong convergence, antithetic version and application to multilevel estimators, Anis Al Gerbi [et al.]	34
Minisymposium: Rare events and stress tests	35
Fluctuation Analysis of Adaptive Multilevel Splitting, Arnaud Guyader	35
Rare event simulation related to financial risks: efficient estimation and sensitivity analysis, Ankush Agarwal [et al.]	35

Rare event probability estimation in the presence of epistemic uncertainty, Math- ieu Balesdent [et al.]	36
Poster session	36
A strong order $1/2$ method for SDEs with discontinuous drift and degenerate diffusion, Gunther Leobacher [et al.]	37
Error Analysis of a Multi-Index Monte Carlo Estimator for a Class of Zakai SPDEs, Zhenru Wang [et al.]	37
Local volatility models enhanced with jumps., Hamza Guennoun [et al.] \ldots	38
Orientational ordering of Janus colloids in cholesteric liquid crystals, Vladimir Rudyak [et al.]	38
Projected adaptive biasing force method: Variance reduction by Helmholtz pro- jection, Houssam Alrachid	39
Friday, July 8th Plenary talks	40 41
Multi-level and Multi-index Monte Carlo, Raul Tempone	41
Higher oder variance reduction for nonlinear Monte Carlo problems, Denis Be- lomestny	42
Multilevel Markov Chain Monte Carlo Methods, Robert Scheichl	42
Minisymposium: Bayesian computation statistics	43
Lossless Bayesian inference in infinite dimension without discretisation or trunca- tion: a case study on Lambda-coalescents, Jere Koskela [et al.]	43
Lossless Bayesian inference in infinite dimension without discretisation or trunca-	
Lossless Bayesian inference in infinite dimension without discretisation or trunca- tion: a case study on Lambda-coalescents, Jere Koskela [et al.]	43

Minisymposium: Mean-fields simulations

	An Augmented Lagrangian Method for Mean Field Type Control with Conges- tion, Mathieu Lauriere [et al.]	45
	Capital distribution in the mean-field Atlas model, Julien Reygner $\ldots \ldots \ldots$	45
	Cross-Dependent Volatility, Julien Guyon	46
Min	isymposium: Stochastic algorithms	46
	Optimism and randomness in linear multi-armed bandit, Alessandro Lazaric [et al.]	46
	Sampling from a strongly log-concave distribution with the Unadjusted Langevin Algorithm, Eric Moulines [et al.]	47
	Pricing American options using martingale bases, Jérome Lelong	48
Min	isymposium: Non linear finance and Nested Monte Carlo	48
	Capital and Funding, Claudio Albanese	48
	Non-parametric regression related to rare-event, using MCMC design, and appli- cation to nested risk computations, Gersende Fort [et al.]	49
	A Dual algorithm for stochastic control problems: Applications to Uncertain Volatility Models and CVA, Pierre Henry-Labordère	49
T • 4		50

List of participants

 $\mathbf{50}$

Tuesday, July 5th

>9:00 (1h)	Coffee welcome		
			> Room Marie Curie
>10:00 (50min)	Plenary talk: Chair: Gil	Peter Glynn les Pages	
>10:50 (50min)	Plenary talk: Chair: Gi	Tong Zhang les Pages	> Amphiteater Pasquier
- 11- 40 (1Emin)			> Amphiteater Pasquier
>11:40 (15min) >11:55 (50min)	D=-		
v11:55 (50min)		Bruno Tuffin no De Marco	
>12:45 (2h)			> Amphiteater Pasquier
			> Room Marie Curie
\$14:45 (1h30)	Unbiased simulation of SDEs Organizer: Aurélien Alfonsi > Amphiteater Pasquier	>14:45 (1h30)	Particle methods Organizer: Robin Ryder > Amphiteater Roussy
>16:15 (30min)			· mpmcater reasy
	Refreshment > Room Marie Curie		
>16:45 (1h30)	Quasi Monte-Carlo Organizer: Nicolas Chopin	>16:45 (1h30)	Model risk and uncertainty Organizer: Claude Martini
	> Amphiteater Pasquier		> Amphiteater Roussy

Plenary talks

Recent Developments in Randomized MLMC

Peter Glynn¹

¹ Stanford University – United States

Multi-level Monte Carlo (MLMC) algorithms have been extensively applied in recent years to obtain schemes that often converge at faster rates than corresponding traditional Monte Carlo methods. In this talk, we shall discuss a randomized method introduced in joint work with Chang-han Rhee, and then describe a stratified alternative estimator. Our principal focus in the talk will be on applications to equilibrium computations for Markov chains, computing value functions, sensitivity estimates, and optimization, and covers joint work with Rhee, Jose Blanchet, and Zeyu Zheng.

Monte Carlo Techniques in Modern Stochastic Optimization for Big Data Machine Learning

Tong Zhang¹

¹ Rutgers University – United States

Many optimization problems encountered in machine learning can be expressed as the minimization of a finite sum of individual loss functions. In recent years, a new class of stochastic optimization methods were developed to solve such problems. These new methods apply variance reduction techniques exited in the Monte Carlo literature to stochastic gradient descent, which lead to significantly faster convergence speed than classical algorithms in optimization. I will present a review of this class of methods, as well as some current directions.

Some applications of importance sampling to dependability analysis

Bruno Tuffin¹

 1 Inria – INRIA – France

We review during this presentation the general principles of Importance Sampling (IS) variance reduction technique and describe how it can be applied to various dependability models. A particular focus will be on our existing works on zero-variance importance sampling approximation techniques applied to this context and its combination with other known variance reduction techniques.

Minisymposium: Unbiased simulation of SDEs

A stochastic parametrix method

Vlad Bally $^{\rm 1}$

 1 Université Paris-Est Marne-la-Vallée – Université de Marne la vallée – France

We represent the semigroup of a diffusion process as the expectation of a random variable which is constructed following the ideas of the parametrix method: this is a development in infinite series of multiple integrals. This idea is combined with a second one which allows us to compute the infinite series using a sample of a Poisson process. The original motivation of such a procedure is to prove regularity for diffusion processes with Hölder continuous coefficients. But this integral representation may be also considered as the starting point of a Monte Carlo procedure. The drawback is that the random variable at hand has infinite variance, and so a direct application is not possible. One has to imagine some complementary truncation procedure in order to get finite variance - and of course this would introduce some bias.

Unbiased simulation of stochastic differential equations using parametrix expansions

Patrik Andersson¹

 1 Uppsala University – Sweden

We consider an unbiased simulation method for multidimensional diffusions based on the parametrix method for solving partial differential equations with Hölder continuous coefficients. This Monte Carlo method which is based on an Euler scheme with random time steps, can be considered as an infinite dimensional extension of the Multilevel Monte Carlo method for solutions of stochastic differential equations with Hölder continuous coefficients. In particular, we study the properties of the variance of the proposed method. In most cases, the method has infinite variance and therefore we propose an importance sampling method to resolve this issue.

Unbiased simulation of stochastic differential equations

Xiaolu Tan $^{\rm 1}$

 1 Ceremade – Université Paris Dauphine - Paris IX – France

We propose an unbiased Monte-Carlo estimator for $\mathbb{E}[g(X_{t_1}, \ldots, X_{t_n})]$, where X is a diffusion process defined by a multi-dimensional stochastic differential equation (SDE). The main idea is to start instead from a well-chosen simulatable SDE whose coefficients are updated at independent exponential times. Such a simulatable process can be viewed as a regime-switching SDE, or as a branching diffusion process with one single living particle at all times. In order to compensate for the change of the coefficients of the SDE, our main representation result relies on the automatic differentiation technique induced by Bismu-Elworthy-Li formula from Malliavin calculus, as exploited by Fournie et al.(1999) for the simulation of the Greeks in financial applications. In particular, this algorithm can be considered as a variation of the (infinite variance) estimator obtained in Bally and Kohatsu-Higa [Section 6.1](2014) as an application of the parametrix method.

Minisymposium: Particle methods

Anytime Monte Carlo

Lawrence Murray¹

¹ Department of Statistics, University of Oxford – United Kingdom

Monte Carlo algorithms typically simulate some fixed number of samples, n, with the real time taken to do so a random variable, T(n). For the purposes of real-time deadlines, particularly in a distributed computing context, an alternative is to fix the real time, t, and allow the number of samples drawn in this time to be a random variable, N(t). Naive estimators constructed from these N(t) samples are not necessarily consistent, however, and in general exhibit length bias with respect to compute time. This talk will introduce a framework for dealing with the length bias for both iid and Markov chain Monte Carlo samplers, and demonstrate the utility of the approach on a large scale sequential Monte Carlo deployment on the Amazon EC2 cloud computing infrastructure.

On the two-filter approximations of marginal smoothing distributions in general state space models

Sylvain Le Corff¹, Eric Moulines², Thi Ngoc Minh Nguyen³

¹ Laboratoire de Mathématiques d'Orsay, Univ. Paris-Sud, CNRS, Université Paris-Saclay. – CNRS : UMR8628, Université Paris Sud - Paris XI, Paris Saclay – France

² Centre de Mathématiques Appliquées - Ecole Polytechnique (CMAP) – Polytechnique - X, CNRS : UMR7641 – CMAP UMR 7641 École Polytechnique CNRS Route de Saclay 91128 Palaiseau Cedex, France

³ Laboratoire Traitement et Communication de l'Information [Paris] (LTCI) – Télécom ParisTech, CNRS : UMR5141 – CNRS LTCI Télécom ParisTech 46 rue Barrault F-75634 Paris Cedex 13, France

The approximation of the smoothing distribution of a state conditional on the observations from the past, the present, and the future is a crucial problem in general state space models. In this talk, we provide a rigorous analysis of such approximations of smoothed distributions provided by the two-filter algorithms. These two-filter approaches combine a forward filter approximating the filtering distributions with a backward information filter approximating a quantity proportional to the posterior distribution of the state given future observations. We extend the results (exponential deviation inequalities, central limit theorems) available for the approximation of smoothing distributions to these procedures and in particular to the proposed methods whose complexity grows linearly with the number of particles.

Variance estimation in the particle filter

Nick Whiteley 1

¹ University of Bristol – United Kingdom

Variance estimation in the particle filter Particle filters provide sampling based approximations of marginal likelihoods and filtering expectations in hidden Markov models. However, estimating the Monte Carlo variance of these approximations, without generating multiple independent realizations of the approximations themselves, is not straightforward. We present an unbiased estimator of the variance of the marginal likelihood approximation, and consistent estimators of the asymptotic variance of the approximations of the marginal likelihood and filtering expectations. These estimators are byproducts of a single run of a particle filter and have no added computational complexity or storage requirements. With additional storage requirements, one can also consistently estimate higher-order terms in the non-asymptotic variance. This is information can be used to approximate the variance-optimal allocation of particle numbers. Joint work with Anthony Lee, University of Warwick.

Minisymposium: Quasi Monte-Carlo

Hermite spaces and QMC methods in quantitative finance

Leobacher Gunther 1

¹ Johannes Kepler University Linz [linz] (JKU) – Altenberger Strasse 69, 4040 Linz, Austria

We present the concept of an Hermite space, which is a special kind of reproducing kernel Hilbert space of functions on the \mathbb{R}^d . We review some tractability results regarding integration in those spaces.

We shall show that those spaces behave nicely with regard to orthogonal transforms of the \mathbb{R}^d ,

which in turn can be shown to correspond to different Brownian path constructions like, e.g., the Brownian bridge construction.

We present a method for finding fast and efficient path constructions for a given pricing problem. Numerical examples will serve to illustrate our findings.

Fast QMC matrix vector multiplication in option pricing

Josef Dick¹, Thong Le Gia¹, Frances Kuo¹, Christoph Schwab²

 1 The University of New South Wales (UNSW) – Australia 2 ETHZ Zurich (ETHZ Zurich) – Switzerland

Quasi-Monte Carlo methods are equal weight quadrature rules used for approximating high dimensional integrals over the unit cube.

In this talk we discuss how quasi-Monte Carlo rules can be modified in certain linear problems to speed up the computation time. The main idea is to design quasi-Monte Carlo rules such that the matrix formed by the quadrature points permits a fast matrix-vector multiplication. This method can be applied to reduce the computational cost in generating normally distributed samples with general covariance matrix. We apply this method to option pricing problems from finance.

This is joint work with Q. T. Le Gia, F. Y. Kuo, Ch. Schwab.

Sequential quasi-Monte Carlo

Nicolas Chopin¹, Mathieu Gerber

¹ ENSAE (ENSAE) – ENSAE ParisTech – France

We derive and study SQMC (Sequential Quasi-Monte Carlo), a class of algorithms obtained by introducing QMC point sets in particle filtering. The complexity of SQMC is $O(N \log N)$, where N is the number of simulations at each iteration, and its error rate is smaller than the Monte Carlo rate $O_P(N^{-1/2})$. The only requirement to implement SQMC is the ability to write the simulation of particle X_t^n given X_{t-1}^n as a deterministic function of X_{t-1}^n and a fixed number of uniform variates. We show that SQMC is amenable to the same extensions as standard SMC, such as forward smoothing, backward smoothing, unbiased likelihood evaluation, and so on. In particular, SQMC may replace SMC within a PMCMC (particle Markov chain Monte Carlo) algorithm. We establish several convergence results. We provide numerical evidence that SQMC may significantly outperform SMC in practical scenarios.

Minisymposium: Model risk and uncertainty

Volatility derivatives and model-free implied leverage

Masaaki Fukasawa ¹

¹ Osaka University – Japan

We revisit robust replication theory of volatility derivatives and introduce a broader class which may be considered as the second generation of volatility derivatives. One of them is a swap contract on the quadratic covariation between an asset price and the model-free implied variance (MFIV) of the asset. It can be replicated in a model-free manner and its fair strike may be interpreted as a model-free measure for the covariance of the asset price and the realized variance. The fair strike is given in a remarkably simple form, which enable to compute it from the Black-Scholes implied volatility surface. We call it the model-free implied leverage (MFIL) and give several characterizations. In particular, we show its simple relation to the Black-Scholes implied volatility skew by an asymptotic method. Further to get an intuition, we demonstrate some explicit calculations under the Heston model. We report some empirical evidence from the time series of the MFIV and MFIL of the Nikkei stock average.

Consistency and model uncertainty in affine interest rate models

Philipp Harms¹

¹ Freiburg University – Germany

Newly arriving market data may be inconsistent with the current model; this is called the recalibration problem. For affine factor models, which are among the most tractable and widely used interest rate models, this is the rule rather than the exception, and one encounters not a risk, but a certainty of model mis-specification. We show in the context of these models that the recalibration problem can be solved by treating model parameters as uncertain and subject to change. Moreover, the model uncertainty can be introduced in a way that preserves the efficiency of Monte Carlo methods. We call the resulting class of "tangent affine" models consistent recalibration (CRC) models.

Kriging of financial term-structures

Areski Cousin¹, Hassan Maatouk², Didier Rullière¹

¹ Institut des Science Financière et d'Assurances (ISFA) – PRES Université de Lyon, Université Claude Bernard - Lyon I – 50 avenue Tony Garnier 69007 Lyon, France

² École Nationale Supérieure des Mines de Saint-Étienne (ENSM-SE) – Groupe des Écoles des Mines (GEM) – 158, Cours Fauriel - 42023 Saint Étienne cedex 2, France

Due to the lack of reliable market information, building financial term-structures may be associated with a significant degree of uncertainty. In this paper, we propose a new term-structure interpolation method that extends classical spline techniques by additionally allowing for quantification of uncertainty. The proposed method is based on a generalization of kriging models with linear equality constraints (market-fit conditions) and shape-preserving conditions such as monotonicity or positivity (no-arbitrage conditions). We define the most likely curve and show how to build confidence bands. The Gaussian process covariance hyper-parameters under the construction constraints are estimated using cross-validation techniques. Based on observed market quotes at different dates, we demonstrate the efficiency of the method by building curves together with confidence intervals for term-structures of OIS discount rates, of zero-coupon swaps rates and of CDS implied default probabilities. We also show how to construct interest-rate surfaces or default probability surfaces by considering time (quotation dates) as an additional dimension.

Wednesday, July 6th

A AA 154				
>9:00 (50min) Plenary talk: Terry Lyons Chair: Francois Delarue				
	> Amphiteater Pasqu			
>9:50 (50min)	Plenary talk: Jean-François Chassagneux Chair: Francois Delarue			
		> Amphiteater Pasquier		
>10:40 (30min)	Coffee break > Room Marie C			
>11:10 (50min)	in) Plenary talk: Michael Ludkovski Chair: Peter Tankov			
		> Amphiteater Pasquier		
+12:00 (2h30) Lunc	h (buffet for registered participants)			
		Poster session		
	Room Marie Curie	> Room Marie Curie		
>14:30 (1h30)	Simulation of BSDEs Organizer: Plamen Turkedjiev	\$14:30 (1h30) Simulation of stochastic graphs and applications Organizer: Florent Benaych-Georges		
	> Amphiteater Pasquier	> Amphiteater Roussy		
>16:00 (30min)				
Refreshment > Room Marie				
>16:30 (1h30)	HPC and GPU Organizer: Lokmane Abbas-Turki	>16:30 (1h30) Optimal trading Organizer: Sophie Laruelle		
	> Amphiteater Pasquier	> Amphiteater Roussy		

Plenary talks

To be announced

Terry Lyons¹

¹ Mathematical Institute [Oxford] (MI) – Mathematical Institute University of Oxford 24-29 St Giles' Oxford, OX1 3LB UK, United Kingdom

Numerical solution of the master equation arising in large population stochastic control

Jean-François Chassagneux 1

¹ Laboratoire de Probabilités et Modèles Aléatoires (LPMA) – Université Paris VII - Paris Diderot – France

In this talk, I will first motivate and describe the master equation introduced in large population stochastic control, which is a PDE written on the Wasserstein space. In particular, I will recall the probabilistic representation of its solution in term of a (fully coupled) FBSDE with McKean-Vlasov interaction. I will then introduce a scheme for this class of BSDEs and demonstrate its convergence both theoretically and practically.

This is a joint work with D. Crisan and F. Delarue.

Stochastic Kriging for Bermudan Option Pricing

Michael Ludkovski¹

¹ UC Santa Barbara (UCSB) – United States

We investigate three new strategies for the numerical solution of optimal stopping problems within the Regression Monte Carlo (RMC) framework of Longstaff and Schwartz. First, we propose the use of stochastic kriging (Gaussian process) meta-models for fitting the continuation value. Kriging offers a flexible, nonparametric regression approach that quantifies approximation quality. Second, we connect the choice of stochastic grids used in RMC to the Design of Experiments paradigm. We examine space-filling and adaptive experimental designs; we also investigate the use of batching with replicated simulations at design sites to improve the signalto-noise ratio. Third, we explore classification models for directly approximating the exercise region. Numerical case studies for valuing Bermudan Puts and Max-Calls under a variety of asset dynamics illustrate that our methods offer significant reduction in simulation budgets over existing approaches.

Minisymposium: Simulation of BSDEs

Simulation of BSDEs with jumps by Wiener chaos expansion

Céline Labart¹, Christel Geiss²

 ¹ Laboratoire de Mathématiques (LAMA) – Université de Savoie, CNRS : UMR5127 – Université de Savoie, UFR SFA Domaine Universitaire, Bâtiment Le Chablais 73376 LE BOURGET DU LAC, France
 ² Department of Mathematics and Statistics - University of Jyväskylä – Department of Mathematics and Statistics P.O.Box 35 (MaD) FI-40014 University of Jyväskylä Finland, Finland

We present an algorithm to solve BSDEs with jumps based on Wiener Chaos Expansion and Picard's iterations. This paper extends the results given in Briand-Labart (2014) to the case of BSDEs with jumps. We get a forward scheme where the conditional expectations are easily computed thanks to chaos decomposition formulas. Concerning the error, we derive explicit bounds with respect to the number of chaos, the discretization time step and the number of Monte Carlo simulations. We also present numerical experiments. We obtain very encouraging results in terms of speed and accuracy.

Adaptive importance sampling in least-squares Monte Carlo algorithms for backward stochastic differential equations

Plamen Turkedjiev¹, Emmanuel Gobet²

 1 King's College London – United Kingdom

² Ecole Polytechnique [Palaiseau] – Ecole Polytechnique – École Polytechnique, 91128 Palaiseau Cedex, France

We design an importance sampling scheme for backward stochastic differential equations (BS-DEs) that minimizes the conditional variance occurring in least-squares Monte Carlo (LSMC) algorithms. The Radon-Nikodym derivative depends on the solution of BSDE, and therefore it is computed adaptively within the LSMC procedure. To allow robust error estimates with respect to the unknown change of measure, we properly randomize the initial value of the forward process. We introduce novel methods to analyze the error: firstly, we establish norm stability results due to the random initialization; secondly, we develop refined concentration-of-measure techniques to capture the variance of reduction. Our theoretical results are supported by numerical experiments.

Numerical approximation of switching problems

Adrien Richou¹, Jean-François Chassagneux

¹ Institut de Mathématiques de Bordeaux (IMB) – Université de Bordeaux – France

We use the representation of Switching Problems as obliquely reflected BSDEs to obtain a discrete time approximation scheme of the solution. We thus focus on the disretization of the obliquely reflected BSDEs. By proving a stability result for the Euler scheme associated to the BSDE, we are able to obtain a rate of convergence in the Lipschitz setting and under the same structural conditions on the generator as the one required for the existence and uniqueness of a solution to the obliquely reflected BSDE. This is a joint work with Jean-François Chassagneux.

Minisymposium: Simulation of stochastic graphs and applications

Community Detection with the Non-Backtracking Operator

Marc Le
large $^{\rm 1}$

 1 inria – INRIA – France

Community detection consists in identification of groups of similar items within a population. In the context of online social networks, it is a useful primitive for recommending either contacts or news items to users. We will consider a particular generative probabilistic model for the observations, namely the so-called stochastic block model and prove that the non-backtracking operator provides a significant improvement when used for spectral clustering.

Default Cascades in Inhomogeneous Financial Networks

Hamed Amini¹, Rama Cont, Andreea Minca

¹ University of Miami – United States

Propagation of balance-sheet or cash-flow insolvency across financial institutions may be modeled as a cascade process on a network representing their mutual exposures. We derive rigorous asymptotic results for the magnitude of contagion in a large financial network and give an analytical expression for the asymptotic fraction of defaults, in terms of network characteristics. Our results extend previous studies on contagion in random graphs to inhomogeneous directed graphs with a given degree sequence and arbitrary distribution of weights. We introduce a criterion for the resilience of a large financial network to the insolvency of a small group of financial institutions and quantify how contagion amplifies small shocks to the network. Our results emphasize the role played by 'contagious links' and show that institutions which contribute most to network instability in case of default have both large connectivity and a large fraction of contagious links. We then allow for random recovery rates for the exposures to defaulted banks and give sufficient conditions such that the size of first order cascade due to contagious links in different networks to be small.

Optimal Control for financial system with default contagion

Agnes Sulem¹, Andreea Minca²

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Our work lies in the area of systemic risk in financial systems and focuses on the study of the tradeoff between financial contagion and benefit of connectivity.

We consider a financial network described as a weighted directed graph, in which nodes represent financial institutions and edges the exposures between them. The distress propagation is modeled as an epidemics on this graph.

We first study the optimal intervention of a lender of last resort who seeks to make equity infusions in a banking system prone to insolvency and to bank runs, under incomplete information of the failure cluster, in order to minimize the contagion effects.

Our study provides some insight on the relation between the value of a financial system, connectivity and optimal intervention.

More precisely, we consider a stylized core-periphery financial network in which links lead to the creation of projects in the outside economy but make banks prone to contagion risk. The controller seeks to maximize, under budget constraints, the value of the financial system defined as the total amount of projects. Under partial information on interbank links, revealed in conjunction with the spread of contagion, the optimal control problem is shown to become a Markov decision problem.

We determine the optimal intervention policy by using dynamic programming. Our results show that the value of the system depends on the connectivity in a non-monotonous way: it first increases with connectivity and then decreases with connectivity.

Moreover, for highly connected systems, it is optimal to increase the rate of intervention in the peripheral banks rather than in core banks.

In the second part of the talk, we study the magnitude of default contagion in a large financial system, in which banks receive benefits from their connections, and investigate how the institutions choose optimally their connectivities by weighting the default risk and the benefits induced by connectivity.

This presentation is based on joint works with Hamed Amini (Miami Univ.), Andrea Minca (Cornell Univ.) and Rui Chen (INRIA/Univ. Paris-Dauphine).

Minisymposium: HPC and GPU

Resolution of a large number of small random symmetric linear systems in single precision arithmetic on GPUs

Lokmane Abbas Turki¹, Stef Graillat²

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This talk will focus on the resolution of a large number of small symmetric linear systems and its parallel implementation on single precision on GPUs. The computations involved by each linear system are independent from the others and the number of unknowns does not exceed 64. For this purpose, we present the adaptation to our context of largely used methods that include: LDLt, Householder reduction to a tridiagonal matrix, parallel cyclic reduction that is not a power of two and the divide and conquer algorithm for tridiagonal eigenproblems. We not only detail the implementation and optimization of each method but we also compare the sustainability of each solution and its performance which include both parallel complexity and cache memory occupation. In the context of solving a large number of small random linear systems on GPU with no information about their conditioning, we show that the best strategy seems to be the use of Householder tridiagonalization + PCR followed if necessary by a divide & conquer diagonalization.

Stratified Nested Regression Monte-Carlo scheme with large scale parallelization

Emmanuel Gobet¹, Jose Lopez-Salas, Plamen Turkedjiev, Carlos Vasquez

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We design a novel algorithm based on Least-Squares Monte Carlo (LSMC) in order to approximate the solution of discrete time dynamic programming equations, like Backward Stochastic Differential Equations (BSDEs). Our algorithm allows massive parallelization of the computations on many core processors such as graphics processing units (GPUs). Our approach consists of a novel method of stratification which appears to be crucial for large scale parallelization. In this way, we minimize the exposure to the memory requirements due to the storage of simulations. Indeed, we note the lower memory overhead of the method compared with previous works.

Toward a benchmark GPU platform to simulate XVA

Babacar Diallo¹

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 6 – France

After the 2007 economic crisis, the calculation of the CVA (Credit Valuation Adjustment) and its extensions XVA (X = C/D/F/T) have become essential. This presentation is devoted to the fast and accurate computation of the XVA using a Nested Monte Carlo (NMC) simulation on GPUs (Graphic Processing Units). The considered model involves lognormal Market factors and uses Dynamic Marshall-Olkin for the Credit ones. The framework however remains quite general and is based on stochastic default intensities using CIR and allows for common-shocks. The complexity of the problem is not only due to the curse of dimensionality, but also to the large number of paths required by NMC. To make the execution time of simulations sufficiently small, a considerable effort was made in order to implement and optimize our code that runs on multiple GPUs.

Minisymposium: Optimal trading

Probability of Fill Estimation Using Trading Signals

Aymen Jedidi¹

 $^1~\mathrm{HSBC}-\mathrm{HSBC}-\mathrm{France}$

In this talk, we present and estimate a model of limit-order execution times using actual limitorder data and survival analysis. We estimate time-to-first-fill and time to completion for buy and sell limit orders, and analyze the effects of explanatory variables such as the limit price, limit size, bid/ask spread, as well as various trading signals such as past returns, order book imbalance, and signed order flow quantity.

Optimal Execution with Statistical Learning

Sophie Laruelle ^{1,2}

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This talk will present how to use statistical learning to build online algorithms to solve different issues encountered in optimal execution. Firstly some aspects of stochastic algorithms will be introduced. Then applications to optimal execution will be developed like finding the optimal posting price, splitting volume across different trading destinations and building intraday volume curves (for implementation shortfall or VWAP).

Quants at work: testing prototypes via Monte Carlo simulations

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¹ Credit Suisse – United Kingdom

In Quantitative Finance, the trading strategies (e.g. investment and execution) are mostly automatized. Before going live with a trading strategy, quants normally build prototypes to test the feasibility of the strategy. On the one hand, they want to be sure that the algorithm behaves as expected e.g. it buys when it has to buy, it sells when it has to sell, and it does not mix Limit Orders and Market Orders. On the other hand, they want to have a first glimpse on the risk-reward profile of the strategy e.g. the distribution of returns.

For these two tests, Monte Carlo simulations are very handy: quants can easily generate prices to (1) check that the algorithm respects the trading rules and (2) study the distribution of returns under different conditions of volatility and market trends. We will try to explain how this testing of prototypes via Monte Carlo simulations is done by some practitioners. In particular, we will illustrate this process with (at least) one example: pairs trading.

Poster session

A Study of the Application of Chaos to the Global Optimization.

Tayeb Hamaizia $^{\rm 1}$

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In this communication we undertake a performance analysis for a new class of evolutionary algorithms called chaos optimization algorithm (COA), recently proposed by Caponetto and al. [1], [2], [3], It was originally proposed to solve nonlinear optimization problems with bounded variables. Dierent chaotic mapping have been considered, combined with several working strategy. In this work, a chaotic strategy is proposed based on a new two-dimensional discrete chaotic attractor.Experiments results showed that the proposed algorithm can achieve good performance.

Hamiltonian Monte Carlo sampling for Wishart distributions with eigenvalue constraints

Alexander Buchholz¹, Nicolas Chopin²

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Sampling from constrained target spaces for Bayesian inference is a non-trivial problem. A recent development has been the use of Hamiltonian Monte Carlo in combination with particle re ection, see [Pakman and Paninski, 2014]. However, Hamiltonian Monte Carlo is sensitive to several hyper parameters, that need to be tuned, to ensure an ecient sampler. For this purpose, [Wang et al., 2013] suggested a black box algorithm that handles this problem. Our approach is to combine the two former ideas to solve the problem of sampling Wishart distributed matrices with eigen-value constraints. Therefore, we exploit the eigenvalue decomposition of positive de nite matrices. The suggested method performs better than the initial sampler of [Everson and Morris, 2000b] when the dimension of the target space grows. Important applications of our sampler are the normal hierarchical model of [Everson and Morris, 2000a] and the rank test in a principal component analysis as in [Choi et al., 2014].

Heterogeneous Risk Preferences in Financial Markets

Tyler Abbot 1

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This paper builds a continuous time model of N heterogeneous agents whose CRRA preferences differ in their level of risk aversion and considers the Mean Field Game (MFG) in the limit as N becomes large. The model represents a natural extension of other work on heterogeneous risk preferences (e.g. Cvitanic, et. al., (2011) "Financial Markets Equilibrium with Heterogeneous Agents". Review of Finance, 16, 285-321) to a continuum of types. I add to the previous literature by characterizing the limit in N and by studying the short run dynamics of the distribution of asset holdings. I find that agents dynamically self select into one of three groups depending on their preferences: leveraged investors, diversified investors, and saving divestors, driven by a wedge between the market price of risk and the risk free rate. The solution is characterized by dependence on individual holdings of the risky asset, which in the limit converge to a stochastic flow of measures. In this way, the mean field is not dependent on the state, but on the control, making the model unique in the literature on MFG and providing a convenient approach for simulation. I simulate by path Monte Carlo both the finite types and continuous types economies and find that both models match qualitative features of real world financial markets. However, the continuous types economy is more robust to the definition of the support of the distribution of preferences and computationally less costly than the finite types economy.

Improved adaptive Multilevel Monte Carlo and applications to finance

Kaouther Hadji¹, Kebaier Ahmed^{2,3,4}, Ben Alaya Mohamed^{3,5}

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 ² University Paris 13 – France
 ³ Laboratory of Excellence MME-DII – France
 ⁴ Chair Risques Financiers – France
 ⁵ University Paris 13 – France

This paper focuses on the study of an original combination of the Euler Multilevel Monte Carlo introduced by M.Giles and the popular importance sampling technique. To compute the optimal choice of the parameter involved in the importance sampling method, we rely on Robbins-Monro type stochastic algorithms. On the one hand, we extend our previous work on adaptive Statistical Romberg method to the Multilevel Monte Carlo setting. On the other hand, we improve it by providing a new adaptive algorithm avoiding the discretization of any additional process. Furthermore, from a technical point of view, the use of the same stochastic algorithms as in our previous work appears to be problematic and we are reduced to employ a specific version of stochastic algorithms with projection (see e.g. Laruelle, Lehalle and Pagès). We firstly prove the almost sure convergence of this stochastic algorithm towards the optimal parameter. Then, we prove a central limit theorem of type Lindeberg-Feller for the new adaptive Euler Multilevel Monte Carlo algorithm together with non-asymptotic Berry-Essen bounds. Finally, we illustrate the efficiency of our method through applications in option pricing for the Heston model.

Parareal methods and applications in finance

Guillaume Sall $^{\rm 1}$

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We aim to show the interest of the parareal method (based on the work of Y. Maday and J.L. Lions) for the approximation of discrete problems based on backward scheme using least-squares regression i.e. the Longstaff-Schwarz algorithm, approximation of a backward stochastic differential equations. We propose a parallelization of the time discretization of the backward dynamic programming principle. It allows us to approximate simultaneously the solution at several time-steps. Here, we analyze this algorithm and the convergence of the parareal scheme. We also give some performance results. Because of its parallel scalability the method is well suited to fast evaluation of CVAs. This is a joint work with G. Pagès and O. Pironneau.

Thursday, July 7th

>9:00 (S0min) Plenary talk: Michael Giles Chair: Benjamin Jourdain				
			> Amphiteater Pasquier	
>9:50 (S0min) Plenary talk: Arnulf Jentzen Chair: Benjamin Jourdain		> Amphiteater Pasquier		
>10:40 (30min)	10:40 (30min) Coffee break		> Room Marie Curie	
>11:10 (50min) Plenary Talk: Barak Pearlmutter Chair: Charles-Albert Lehalle				
>12:00 (2h30)			> Amphiteater Pasquier	
	buffet for registered participants)			
		>13:30 (1h)	Poster session	
	> Room Marie Curie		> Room Marie Curie	
>14:30 (1h30)		>14:30 (1h30)	CDF and a simulation	
	ent computations of sensitivities rganizer: Charles-Albert Lehalle		SDE approximation Organizer: Emmanuelle Clément	
	> Amphiteater Pasquier		> Amphiteater Roussy	
>16:00 (30min)	Refres	hment	> Room Marie Curie	
F16:30 (1h30)	Multilevel Monte Carlo Organizer: Ahmed Kebaier	>16:30 (1h30)	Rare events and stress tests Organizer: Ludovic Goudenège	
	> Amphiteater Pasquier		> Amphiteater Roussy	

Plenary talks

Two new MLMC applications

Michael Giles¹

 1 University of Oxford – United Kingdom

This talk will discuss two new MLMC applications. The first is a collaboration with Kavita Ramanan (Brown University) on multi-dimensional reflected diffusions. The key issue is a poor strong convergence due to the treatment of the reflection at the boundary. This is addressed through adaptive time-stepping, reducing the timestep size near the boundary in a way which improves the accuracy without substantially increasing the cost. The second is a collaboration with Howard Thom (University of Bristol) on EVPI (Expected Value of Partial Information). This involves a nested simulation, and an antithetic treatment is used to improve the variance of the multilevel estimator. In both cases I will present some numerical results and outline the supporting numerical analysis.

Nonlinear stochastic ordinary and partial differential equations: regularity properties and numerical approximations

Arnulf Jentzen ¹

¹ Department Mathematics ETHZ – Switzerland

In this talk we present a few recent results on regularity properties and numerical approximations for stochastic ordinary and partial differential equations with non-globally monotone nonlinearities. In particular, we establish strong convergence rates for Cox-Ingersoll-Ross (CIR) processes, stochastic Duffing-van der Pol oscillators, stochastic Lorenz equations, and Cahn-Hilliard-Cook equations. CIR processes are widely used in the financial engineering industry to estimate prices of financial derivatives. We also present a calibration result for CIR processes and stocks from the S & P 500 (Standard & Poor's 500) stock market index. The talk is based on joint works with Martin Hairer, Martin Hutzenthaler, Thomas Mueller-Gronbach, Marco Noll, and Larisa Yaroslavtseva.

More details on this topic can also be found at https://www.math.ethz.ch/sam/research/projects. html?details=33.

Automatic Differentiation in Machine Learning: Survey and Revisionist History

Barak Pearlmutter ¹

¹ Maynooth University – Department of Computer Science, Maynooth University, Maynooth, Co. Kildare, Ireland

Automatic Differentiation (AD) is the mechanical transformation of computer code implementing a numeric function to also exactly and efficiently calculate various derivatives. AD had its first successes at the dawn of the computer age, and has continued to develop. In this talk we will explain the basic ideas of AD. We will also explore where it can be of assistance in rapidly and correctly implementing and analyzing complicated models, and where manual assistance is still required to attain desirable efficiencies.

Minisymposium: Efficient computations of sensitivities

Automatic Differentiation for Complex options and algorithms

Olivier Pironneau $^{\rm 1}$

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Automatic differentiation of computer programs gives an exact evaluation of sensitivities ; there are two approaches, the so called direct and reverse modes. Both are callable by linking C++ librairies based on operator overloading.

In this talk we shall present the method, discuss the computing cost and when to use the direct or reverse mode.

We will also discuss the use of automatic differentiation together with multilevel Monte-Carlo.

Practical implementation of adjoint algorithmic differentiation (AAD)

Sébastien Geeraert¹

¹ Murex Analytics – Murex – France

Adjoint algorithmic differentiation (AAD) is a method producing the exact derivatives of a computation, in an automatic and efficient way. By using pathwise differentiation with AAD, we can compute Monte Carlo sensitivities of a price, in a time that does not depend on the number of sensitivities. We first show a naive implementation in C++, working on classical processors (CPU) or on graphic cards (GPU). We are confronted with a major drawback of AAD: every intermediate result must be saved, which leads to a large memory consumption. We then present a few possible optimizations to increase parallelization and limit memory consumption, and their practical effects on an example of CVA computation. Thanks to these optimizations, we achieve interesting gains in comParison to finite differences. Lastly, we mention possible ways to handle non-smooth payoffs, such as Vibrato Monte Carlo.

AAD applications in Finance

Adil Reghai¹

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AAD is a powerful tool that is progressively used in the banking industry solving huge computational burden due an evolving regulation. In this presentation we present AAD and the many applications identified in Finance and how this new technique is reshaping the computational world in finance.

Minisymposium: SDE approximation

Rate of convergence of the Euler scheme for SDE with discontinuous drift

Antoine Lejay 1

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During this talk, we present some results concerning the weak rate of convergence of the Euler scheme for SDE with discontinuous drift. For this, we present a general approach which combined considerations on the class of regularity of drifts as well as the class of terminal conditions. From this, using Girsanov theorem, Malliavin calculus and analysis of PDE, several rates could be deduced. The sub-optimality of some rates of also discussed.

Collaboration with Arturo Kohatsu-Higa (Ritsumeikan University, Kyoto, Japan) and Kazuhiro Yasuda (Hosei University, Tokyo).

Trajectorial coupling between one-dimensional diffusions with linear diffusion coefficient and their Euler scheme

Arnaud Gloter ¹, Emmanuelle Clément ²

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It is well known that the strong error approximation, in the space of continuous paths equipped with the supremum norm, between a diffusion process, with smooth coefficients, and its Euler approximation with step 1/n is $O(n^{-1/2})$ and that the weak error estimation between the marginal laws, at the terminal time T, is $O(n^{-1})$. In this talk, we study the *p*-Wasserstein distance between the law of the trajectory of a diffusion process, with linear diffusion coefficient, and its Euler scheme. Using the Komlòs, Major and Tusnády construction, we show that this Wasserstein distance is of order $\log n/n$.

A Symmetrized Milstein scheme with strong rate of convergence for some CEV-like SDEs

Mireille Bossy¹

 1 INRIA – INRIA – France

We consider the approximation problem of SDE with non-Lipschitz diffusion coefficients. More specifically, we consider diffusion coefficients of the typical form $|x|^a$, used in popular volatility models in finance such as CEV models. In the context of a one dimensional SDE, we present a modified explicit Milstein scheme that allows us to prove strong convergence at rate one under some theoretical restrictions on the drift and diffusion parameters. The proof lies on classical arguments, except for the treatment of the local error that relies on a priori analysis of a weighted local error. If the theoretical rate one is optimal, as for others approximation strategies the theoretical restrictions imposed in the proof are often pessimistic as shown by some comparative numerical experiments.

This is a joint work with Hector Olivero (Universitad de Chile).

Minisymposium: Multilevel Monte Carlo

General multilevel adaptations for stochastic approximation algorithms

Steffen Dereich¹, Thomas Müller-Gronbach²

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² Universität Passau – Germany

Stochastic approximation algorithms are a standard tool for the numerical computation of zeroes of functions $f : \mathbb{R}^d \to \mathbb{R}^d$ of the form $f(\theta) = \mathbb{E}[F(\theta, U)]$ with U denoting a random variable and F an appropriate measurable function.

Robbins and Monro introduced in 1951 a dynamical system that converges to solutions and is based on independent realisations of U (under appropriate assumptions). Research on this topic remained active and resulted in a variety of results. A natural question was whether the approach can be combined with the recently discovered multilevel paradigm in the case where $F(\theta, U)$ is not simulatable. Indeed, as shown by N. Frikha the combination bears potential and leads to new efficient schemes in the context of SDEs.

In this talk we provide new multilevel stochastic approximation algorithms and present complexity theorems on L^p -errors in the spirit of the original work of M. Giles on multilevel Monte Carlo. In contrast to previous work, our error analysis requires significantly weaker assumptions which makes it applicable in a wide variety of examples.

Multilevel Monte Carlo for McKean-Vlasov SDEs

Lukasz Szpruch¹

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Stochastic Interacting Particle System (SIPS) and they limiting stochastic McKean-Vlasov equations offer a very rich and versatile modelling framework. On one hand interactions allow us to capture complex dependent structure, on the other provide a great challenge for Monte Carlo simulations. The non-linear dependence of the approximation bias on the statistical error makes classical variance reduction techniques fail in this setting. In this talk, we will devise a strategy that will allow us to overcome this difficulty. In particular, we will establish Multilevel Monte Carlo estimator for SIPS and demonstrate it computational superiority over standard Monte Carlo techniques.

Ninomiya-Victoir scheme: strong convergence, antithetic version and application to multilevel estimators

Anis Al Gerbi¹, Benjamin Jourdain¹, Emmanuelle Clément²

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In this paper, we are interested in the strong convergence properties of the Ninomiya-Victoir scheme which is known to exhibit weak convergence with order 2. We prove strong convergence

with order 1/2. This study is aimed at analysing the use of this scheme either at each level or only at the finest level of a multilevel Monte Carlo estimator: indeed, the variance of a multilevel Monte Carlo estimator is related to the strong error between the two schemes used on the coarse and fine grids at each level. Recently, Giles and Szpruch proposed a scheme permitting to construct a multilevel Monte Carlo estimator achieving the optimal complexity $O(\epsilon - 2)$ for the precision ϵ . In the same spirit, we propose a modified Ninomiya-Victoir scheme, which may be strongly coupled with order 1 to the Giles-Szpruch scheme at the finest level of a multilevel Monte Carlo estimator. Numerical experiments show that this choice improves the efficiency, since the order 2 of weak convergence of the Ninomiya-Victoir scheme permits to reduce the number of discretization levels.

Minisymposium: Rare events and stress tests

Fluctuation Analysis of Adaptive Multilevel Splitting

Arnaud Guyader 1

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Multilevel Splitting, also called Subset Simulation, is a Sequential Monte Carlo method to simulate realisations of a rare event as well as to estimate its probability. This talk is concerned with the convergence and the fluctuation analysis of Adaptive Multilevel Splitting techniques. In contrast to their fixed level version, adaptive techniques estimate the sequence of levels on the fly and in an optimal way, with only a low additional computational cost. However, very few convergence results are available for this class of adaptive branching models, mainly because the sequence of levels depends on the occupation measures of the particle systems. We will present the consistency of these methods as well as a central limit theorem. In particular, we show that the precision of the adaptive version is the same as the one of the fixed-levels version where the levels would have been placed in an optimal manner.

Rare event simulation related to financial risks: efficient estimation and sensitivity analysis

Ankush Agarwal¹, Stefano De Marco¹, Emmanuel Gobet¹, Gang Liu¹

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We propose an adaptive rare event simulation method based on reversible shaking transformations on path space to estimate the rare event statistics arising in different financial risk settings which are embedded within a unified framework of isonormal Gaussian process. We prove the convergence of the adaptive algorithm and provide key theoretical results on the accompanying Markov chain sampler. We tackle the important problem of calculating sensitivities of rare event statistics with respect to the model parameters by providing general analytical formulas. We demonstrate the strength of our method and application of our results in various numerical examples which cover usual semi-martingale stochastic models (not necessarily Markovian) driven by Brownian motion and, also, models driven by fractional Brownian motion (non semi-martingale) to address various financial risks.

Rare event probability estimation in the presence of epistemic uncertainty

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The accurate estimation of rare event probabilities is a crucial problem in engineering to characterize the reliability of complex systems. Several methods such as Importance Sampling or Subset Sampling have been proposed to perform the estimation of such events more accurately (i.e. with a lower variance) than crude Monte-Carlo method. However, these methods often do not take into account epistemic uncertainty either on the probability distributions of the input variables either on the limit state function modeling. Aleatory uncertainties are usually described through probabilistic formalism and epistemic uncertainties with using intervals. Such problems typically induce intricate optimization and numerous probability estimations in order to determine the upper and lower bounds of the probability estimate. The calculation of these bounds is often numerically intractable for rare event probability, due to the high computational cost required. In this presentation, two methodologies are described in order to handle both types of uncertainty with reduced simulation budget. These methods combine rare event simulation algorithms (Importance Sampling and Subset Simulation), Kriging-based surrogate model of the simulation code, and optimization process. To reduce the simulation cost, dedicated refinement strategies of the surrogate model are proposed taking into account the presence of both aleatory and epistemic uncertainties. The efficiency of the proposed approaches, in terms of accuracy of the found results and computational cost, is assessed on academic and launch vehicle stage fallout test cases.

Poster session

A strong order 1/2 method for SDEs with discontinuous drift and degenerate diffusion

Gunther Leobacher¹, Michaela Szölgyenyi²

 1 Johannes Kepler University Lin
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When solving certain stochastic optimization problems, e.g., in mathematical finance, the optimal control policy is of threshold type, meaning that it depends on the controlled process in a discontinuous way. The stochastic differential equations (SDEs) modeling the underlying process then typically have discontinuous drift and degenerate diffusion parameter. This motivates the study of a more general class of such SDEs. We prove an existence and uniqueness result, based on certain a transformation of the state space by which the drift is "made continuous". As a consequence the transform becomes useful for the construction of a numerical method. The resulting scheme is proven to converge with strong order 1/2. This is the first result of that kind for such a general class of SDEs. In examples we show the necessity of the geometric conditions we pose.

Error Analysis of a Multi-Index Monte Carlo Estimator for a Class of Zakai SPDEs

Zhenru Wang¹, Christoph Reisinger¹

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In this article, we propose a space-time Multi-Index Monte Carlo estimator for a one-dimensional parabolic stochastic partial differential equation (SPDE) of Zakai type. We compare the complexity with the Multilevel Monte Carlo method of Giles and Reisinger (2012), and find, by means of Fourier analysis, that the MIMC method i) has suboptimal complexity of $\varepsilon^{-2}(\log \varepsilon)^2$ for RMSE ε if the same spatial discretisation as in the MLMC method is used, ii) has the same optimal complexity as MLMC of ε^{-2} if a carefully adapted discretisation is used, and iii) does not necessarily fit into the standard MIMC analysis framework of Haji-Ali et al. (2015) for non-smooth functionals. Numerical tests confirm these findings empirically.

Local volatility models enhanced with jumps

Hamza Guennoun ¹, Pierre Henry-Labordère ²

 1 Société Générale – France 2 Société Générale - CIB – France

In this paper, we study the calibration to market call prices $C^{mkt}(t, K)$ of a local volatility model enhanced with jumps. Instead of giving an exact calibration condition on the local volatility, we introduce an approximate process S_t^{ϵ} satisfying a well-defined nonlinear McKean SDE driven by a Cox process, such that $\mathbb{E}[S_t^{\epsilon} - K)_+]$ converges to $C^{mkt}(t, K)$ as ϵ goes to 0 for all (t, K). This implies that the particle method, applied to the process S_t^{ϵ} , which is used for the calibration of the local volatility, converges numerically. We illustrate the accuracy of our calibration algorithm with various numerical experiments. Finally, we extend this model by allowing jump in the local volatility.

Orientational ordering of Janus colloids in cholesteric liquid crystals

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Solutions of colloidal particles in liquid crystal (LC) materials usually demonstrate unique properties related, in particular, to the self-assembly of colloidal particles in LCs. Complex inter-particle and particle–LC interactions lead to the complex special particle assemblies in both nematic and cholesteric LCs due to the formation of LC defects. In the simplest case of a single spherical particle with strong homeotropic anchoring in a nematic LC, the overall topological charge leads to the formation of defects around the particle (so-called "Satellite point" and "Saturn ring" structures). In cholesteric LC media, the "Saturn ring" defect twists around the particle, forming a number of turns depending on cholesteric pitch. A complex asymmetric shape of these defects allows the particles to be aligned by LC media.

We used the extended Frank elastic continuum approach with Monte-Carlo Metropolis annealing optimization to estimate energy profiles of Janus particles of various sizes in cholesteric media. The special form of grid approximation for Frank elastic continuum theory was used for highperformance GPU parallelization. Calculations have shown the possibility of strong orientational ordering of particles in such systems, especially for large particles. The effect can be controlled by the adjusting cholesteric pitch. The effect can potentially be used in materials for electronic paper. Its obvious advantage is that the switching time between the described states is limited only by the rotation of the spherical particles and should be smaller than in conventional e-paper, where it is limited by the translational motion of particles through the cell. This work was partially supported by RFBR, research project No. 15-59-32410, 15-02-08269, 15-32-20859.

Projected adaptive biasing force method: Variance reduction by Helmholtz projection

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In computational statistical physics, good sampling techniques are required to obtain macroscopic properties through averages over microscopic states. The main difficulty is that these microscopic states are typically clustered around typical configurations, and a complete sampling of the configurational space is thus typically very complex to achieve. Techniques have been proposed to efficiently sample the microscopic states in the canonical ensemble. An important example of quantities of interest in such a case is the free energy. Free energy computation techniques are very important in molecular dynamics computations, in order to obtain a coarse-grained description of a high-dimensional complex physical system.

We explore an extension of the classical adaptive biasing force (ABF) technique, which is used to compute the free energy associated to the Boltzmann-Gibbs measure and a reaction coordinate function. The problem of this method is that the approximated gradient of the free energy, called biasing force, is not a gradient. The contribution to this field is to project the estimated biasing force on a gradient using the Helmholtz decomposition. In practice, the new gradient force is obtained by solving Poisson problem. Using entropy techniques, we study the longtime behavior of the nonlinear Fokker-Planck equation associated with the ABF process. We prove exponential convergence to equilibrium of the estimated free energy, with a precise rate of convergence in function of the Logarithmic Sobolev inequality constants of the canonical measure conditioned to fixed values of the reaction coordinate. The interest of this projected ABF method compared to the original ABF approach is that the variance of the new biasing force is smaller, which yields quicker convergence to equilibrium.

Friday, July 8th

A AA 18A			
>9:00 (50min)	Plenary talk: Raul Tempone Chair: Idriss Kharroubi		
			> Amphiteater Pasquier
>9:50 (50min)	Plenary talk: Denis Belomestny Chair: Idriss Kharroubi		
>10:40 (30min)			> Amphiteater Pasquier
*10:40 (30min)	Coffee break		> Room Marie Curie
>11:10 (50min)	Plenary talk: Robert Scheichl Chair: Noufel Frikha		
			> Amphiteater Pasquier
>12:00 (1h30) Lunch (buffet for registered participants)			
			> Room Marie Curie
>13:30 (1h30)	Bayesian computation statistics Organizer: Christian Robert		Mean-fields simulations Organizer: Francois Delarue
	Amphiteater Pasquier		> Amphiteater Roussy
>15:00 (30min)	Refreshment > Room		> Room Marie Curie
>15:30 (1h30)	Stochastic algorithms Organizer: Jerome Lelong		r finance and Nested Monte Carlo Organizer: Stephane Crepey
	> Amphiteater Pasquier		> Amphiteater Roussy

Plenary talks

Multi-level and Multi-index Monte Carlo

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We will first discuss adaptive strategies in the context of Multilevel Monte Carlo (MLMC) Methods for (i) Itô Stochastic Differential Equations, (ii) Stochastic Reaction Networks modeled by Pure Jump Markov Processes and (iii) Partial Differential Equations with random inputs. Here, the notion of adaptivity includes several aspects such as mesh refinements based on either a priori or a posteriori error estimates, the local choice of different time stepping methods and the selection of the total number of levels and the number of samples at different levels. Our Adaptive MLMC estimator uses a hierarchy of adaptively refined, non-uniform time discretizations, and, as such, it may be considered a generalization of the uniform discretization MLMC method introduced independently by M. Giles and S. Heinrich. In particular, we show that our adaptive MLMC algorithms are asymptotically accurate and have the correct complexity with an improved control of the multiplicative constant factor in the asymptotic analysis. We also developed techniques for estimation of parameters needed in our MLMC algorithms, such as the variance of the difference between consecutive approximations. These techniques take particular care of the deepest levels, where for efficiency reasons only a few realizations are available to produce essential estimates. Moreover, we show the asymptotic normality of the statistical error in the MLMC estimator, justifying in this way our error estimate that allows prescribing both the required accuracy and confidence level in the final result.

In the second part of this talk, we describe and analyze the Multi-Index Monte Carlo (MIMC) for computing statistics of the solution of a PDE with random data. MIMC is both a stochastic version of the combination technique introduced by Zenger, Griebel and collaborators and an extension of the Multilevel Monte Carlo (MLMC) method first described by Heinrich and Giles. Instead of using first-order differences as in MLMC, MIMC uses mixed differences to reduce the variance of the hierarchical differences dramatically. These mixed differences yield new and improved complexity results, which are natural generalizations of Giles's MLMC analysis, and which increase the domain of problem parameters for which we achieve the optimal convergence. We finally show the effectiveness of MIMC in some computational tests, including PDEs with random coefficients and Stochastic Particle Systems.

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"A Continuation Multilevel Monte Carlo algorithm", by N. Collier, A.-L. Haji-Ali, F. Nobile,
 E. von Schwerin and R. Tempone. BIT Numer. Math., 55, pp. 399–432, 2015.

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"Multi-Index Stochastic Collocation for random PDEs", by A. L. Haji Ali, F. Nobile, L. Tamellini and R. Tempone. To appear in Computers and Mathematics with Applications, 2016.
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Higher oder variance reduction for nonlinear Monte Carlo problems

Denis Belomestny¹

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In this talk I present several new variance reduction approaches which are applicable to a variety of challenging nonlinear Monte Carlo problems like simulation-based optimal stopping or simulation of McKean-Vlasov-type equations. A distinctive feature of these approaches is that they allow for a significant reduction of the computational complexity not only up to a constant but also in order. A numerical performance of these approaches will also be illustrated.

Multilevel Markov Chain Monte Carlo Methods

Robert Scheichl¹

¹ University of Bath (UK) – United Kingdom

In this talk I will address the prohibitively large cost of Markov chain Monte Carlo methods

for uncertainty quantification in large-scale PDE applications with high (or infinite) dimensional parameter spaces. We propose a new multilevel Metropolis-Hastings algorithm, and give an abstract theorem on its cost. We then provide a detailed analysis of the assumptions in the theorem for a typical model problem in subsurface flow, and show gains of at least one order in the ϵ -cost over standard Metropolis-Hastings both theoretically and numerically. This is joint work with T. Dodwell (Exeter), C. Ketelsen (Boulder) and A. Teckentrup (Warwick).

Minisymposium: Bayesian computation statistics

Lossless Bayesian inference in infinite dimension without discretisation or truncation: a case study on Lambda-coalescents

Jere Koskela¹, Paul Jenkins¹, Dario Spano¹

¹ University of Warwick – United Kingdom

In this talk I will introduce the class of Lambda-coalescents, which are naturally parametrised by infinite dimensional probability measures. I will show that these measures can be consistently inferred from data without discretising or truncating the problem under verifiable conditions on the prior. The method resembles the so-called Likelihood Informed Subspaces approach to Bayesian inverse problems under Gaussian measures, and we expect it to generalis beyond just the Lambda-coalescent setting without difficulty. I will conclude by providing an empirical comParison of noisy and exact pseudo-marginal MCMC algorithms for sampling the resulting posteriors.

Adaptive, delayed-acceptance MCMC for targets with expensive likelihoods

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When conducting Bayesian inference, delayed acceptance (DA) Metropolis-Hastings (MH) algorithms and DA pseudo-marginal MH algorithms can be applied when it is computationally expensive to calculate the true posterior or an unbiased estimate thereof, but a computationally cheap approximation is available. A first accept-reject stage is applied, with the cheap approximation substituted for the true posterior in the MH acceptance ratio. Only for those proposals which pass through the first stage is the computationally expensive true posterior (or unbiased estimate thereof) evaluated, with a second accept-reject stage ensuring that detailed balance is satisfied with respect to the intended true posterior. In some scenarios there is no obvious computationally cheap approximation. A weighted average of previous evaluations of the computationally expensive posterior provides a generic approximation to the posterior. If only the k-nearest neighbours have non-zero weights then evaluation of the approximate posterior can be made computationally cheap provided that the points at which the posterior has been evaluated are stored in a multi-dimensional binary tree, known as a KD-tree. The contents of the KD-tree are potentially updated after every computationally intensive evaluation. The resulting adaptive, delayed-acceptance [pseudo-marginal] Metropolis-Hastings algorithm is justified both theoretically and empirically. Guidance on tuning parameters is provided and the methodology is applied to a discretely observed Markov jump process characterising predator-prey interactions and an ODE system describing the dynamics of an autoregulatory gene network.

Sequential Monte Carlo with estimated likelihoods

Richard Everitt¹

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The development of exact approximate Monte Carlo methods, in which unbiased estimates of densities are used within Markov chain Monte Carlo (MCMC) or sequential Monte Carlo (SMC) algorithms without loss of exactness, is one of the most important recent innovations in the field. This talk concerns the use of both exact approximations and inexact approximations or "noisy" methods (where low variance alternatives to unbiased approximations are used instead) within importance sampling and SMC algorithms. In noisy methods the exactness of the algorithm is lost, but in some cases this proves to be insignificant compared to computational savings or improved variance of estimates produced by finite runs. In the context of many applied researchers accepting the use of other approximate methods (such as approximate Bayesian computation) further investigation of the use of noisy methods is warranted.

Minisymposium: Mean-fields simulations

An Augmented Lagrangian Method for Mean Field Type Control with Congestion

Mathieu Lauriere¹, Yves Achdou¹

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The theory of mean field type control aims at describing the behaviour of a large number of interacting agents using a common feedback. A type of problems that have recently raised a lot of interest concerns congestion effects. These problems model situations in which the cost of displacement of the agents increases in regions where the density is large (as, for instance, in crowd motion). We obtain existence and uniqueness of suitably defined weak solutions for a system of partial differential equations arising in this setting. The solutions are characterized as the optima of two optimal control problems in duality. To solve numerically this problem, a monotone finite difference scheme is proposed and shown to have a variational interpretation. Then an augmented Lagrangian is defined and an Alternating Direction Method of Multipliers is proposed for solving the latter variational problem. Two kinds of boundary conditions are considered: periodic conditions and boundary conditions associated to state constrained problems which allows one to model more realistic situations. Various test cases and numerical results are presented.

Capital distribution in the mean-field Atlas model

Julien Reygner 1

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Atlas models are a class of equity market models in which the dynamics of the price of an asset only depends on its rank within the portfolio. When this dynamics exhibits a mean-field

scaling, propagation of chaos techniques arising from kinetic theory lead to a functional nonlinear description of the evolution of the market. In particular, one can obtain a fluid limit for the distribution of the capital, which reproduces some features of actual data, such as a Pareto law for capital concentration.

Cross-Dependent Volatility

Julien Guyon ^{1,2,3}

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Local volatilities in multi-asset models typically have no cross-asset dependency. In this talk, we propose a general framework for pricing and hedging derivatives in cross-dependent volatility (CDV) models, i.e., multi-asset models in which the volatility of each asset is a function of not only its current or past levels, but also those of the other assets. For instance, CDV models can capture that stock volatilities are driven by an index level, or recent index returns. We explain how to build all the CDV models that are calibrated to all the asset smiles, solving in particular the longstanding smiles calibration problem for the "cross-aware" multidimensional local volatility model. CDV models are rich enough to be simultaneously calibrated to other instruments, such as basket smiles, and we show that the model can fit a basket smile either by means of a correlation skew, like in the classical "cross-blind" multi-asset local volatility model, or using only the cross-dependency of volatilities itself, in a correlation-skew-free model, thus proving that steep basket skews are not necessarily a sign of correlation skew. We can even calibrate CDV models to basket smiles using correlation skews that are opposite to the ones generated by the classical cross-blind models, e.g., calibrate to large negative index skews while requiring that stocks are less correlated when the market is down. All the calibration procedures use the particle method; the calibration of the implied "local in basket" CDV uses a novel fixed point-compound particle method. Numerical results in the case of the FX smile triangle problem illustrate our results and the capabilities of CDV models.

Minisymposium: Stochastic algorithms

Optimism and randomness in linear multi-armed bandit

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In this talk I will review the linear multi-armed bandit problem and discuss two main algorithmic design approaches. The first one relies on the celebrated principle of "optimism in face of uncertainty", which prescribes to select at each step the action that maximizes the performance when the problem parameters are optimistically chosen within the current level of uncertainty. While the resulting algorithms are effective from a learning point of view, they are computationally expensive as the number of available actions increases. An alternative approach is to use a randomized algorithm, the so-called Thompson sampling, which drastically reduces the computational complexity. On the other hand, this poses theoretical challenges to "control" the negative effects of the stochastic choice of actions. I will indeed show that even in this case it is crucial to preserve a minimum level of optimism to guarantee an order-optimal performance. Finally, I will discuss possible extensions beyond the linear bandit problem.

Sampling from a strongly log-concave distribution with the Unadjusted Langevin Algorithm

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We consider in this paper the problem of sampling a probability distribution π having a density w.r.t. the Lebesgue measure on \mathbb{R}^d , known up to a normalisation factor $x \mapsto e^{-U(x)} / \int_{\mathbb{R}^d} e^{-U(y)} dy$. Under the assumption that U is continuously differentiable, ∇U is globally Lipschitz and Uis strongly convex, we obtain non-asymptotic bounds for the convergence to stationarity in Wasserstein distances of the sampling method based on the Euler discretization of the Langevin stochastic differential equation for both constant and decreasing step sizes. The dependence on the dimension of the state space of the obtained bounds is studied to demonstrate the applicability of this method in the high dimensional setting. The convergence of an appropriately weighted empirical measure is also investigated and bounds for the mean square error and exponential deviation inequality for Lipschitz functions are reported. Some numerical results are presented to illustrate our findings.

Pricing American options using martingale bases

Jérome Lelong¹

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In this work, we propose an algorithm to price American options by directly solving the dual minimization problem introduced by Rogers [2002]. Our approach relies on approximating the set of uniformly square integrable martingales by a finite dimensional Wiener chaos expansion. Then, we use a sample average approximation technique to efficiently solve the optimization problem. Unlike all the regression based methods, our method can transparently deal with path dependent options without extra computations and a parallel implementation writes easily with very little communication and no centralized work. We test our approach on several multi–dimensional options with up to 40 assets and show the impressive scalability of the parallel implementation.

Minisymposium: Non linear finance and Nested Monte Carlo

Capital and Funding

Claudio Albanese $^{\rm 1}$

¹ Global Valuation – United Kingdom

In the aftermath of the financial crisis, regulators launched in a major effort of banking reform aimed at securing the financial system by raising collateralisation and capital requirements. Notwithstanding finance theories according to which costs of capital and of funding for collateral are irrelevant to decisions, banks have introduced an array of XVA metrics to precisely quantify them. In particular, KVA (capital valuation adjustment) and FVA (funding valuation adjustment) are emerging as metrics of key relevance.

In this paper we frame XVA metrics within a consistent model for the capital structure of a bank. We do not postulate that markets for contingent claims are complete. The fact that a bank is intrinsically leveraged, invalidates several of the conclusions of Modigliani-Miller theory but not all. We introduce a framework for assessing KVA, reflect it into entry prices and distribute it gradually to the bank's shareholders through a dividend policy that would be sustainable even in the limit case of a portfolio held on a run-off basis, with no new trades ever entered in the future. Our FVA is defined asymmetrically since there is no benefit in holding excess capital in the future. We notice that capital is fungible as a source of funding for variation margin (but not for initial margin), causing a material reduction in the FVA numbers.

Non-parametric regression related to rare-event, using MCMC design, and application to nested risk computations

Gersende Fort¹, Emmanuel Gobet², Eric Moulines³

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We consider the problem of estimating the mean of a function of a conditional expectation in a rare-event regime, using Monte Carlo simulations. This is a problem of nested Monte Carlo computations with a special emphasis on the distribution tails. A major application is the risk management of portfolios written with derivative options; these computations are also an essential concern for Solvency Capital Requirement in insurance. In our approach, the outer expectation is evaluated using a Metropolis-hastings type algorithm able to suitably sample the rare-event scenarii. The inner expectation is computed using non-parametric regression tools, in a context of non i.i.d. design. We provide some non-asymptotic quadratic error bounds and we experiment the final algorithm on financial examples.

A Dual algorithm for stochastic control problems: Applications to Uncertain Volatility Models and CVA

Pierre Henry-Labordère 1

 1 Société Générale - CIB – France

We derive an algorithm in the spirit of Rogers and Davis, Burstein that leads to upper bounds for stochastic control problems. Our bounds complement lower biased estimates recently obtained in Guyon, Henry-Labordere. We evaluate our estimates in two numerical examples motivated from mathematical finance.

Joint work with Christian Litterer and Zhenjie Ren.

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