Hermite spaces and QMC methods in quantitative finance

Gunther Leobacher

Johannes Kepler University Linz (JKU)

International Conference on Monte Carlo techniques
Paris, July 5–8, 2016

Outline of the talk

1. Derivative pricing
2. QMC methods
3. Generation of Brownian paths
4. Hermite spaces

Outline of the talk

1. Derivative pricing
2. QMC methods
3. Generation of Brownian paths
4. Hermite spaces

Outline of the talk

1. Derivative pricing
2. QMC methods
3. Generation of Brownian paths
4. Hermite spaces

Outline of the talk

1. Derivative pricing
2. QMC methods
3. Generation of Brownian paths
4. Hermite spaces

BS and SDE models

Consider SDE-model ($m+1$-dimensional)

$$
\begin{aligned}
dS_t &= b(t, S_t)dt + a(t, S_t)dW_t\,,\ \ t \in [0, T],\\
S_0 &= s_0
\end{aligned}
$$

BS and SDE models

Consider SDE-model ($m+1$-dimensional)

$$dS_t = b(t, S_t)dt + a(t, S_t)dW_t\,,\ \ t \in [0, T],$$
$$S_0 = s_0$$

S^0 ... riskless asset

BS and SDE models

Consider SDE-model ($m+1$-dimensional)

$$dS_t = b(t, S_t)dt + a(t, S_t)dW_t\,,\ \ t \in [0, T],$$
$$S_0 = s_0$$

S^0 ... riskless asset $S^1, \ldots, S^k$ risky assets

BS and SDE models

Consider SDE-model ($m+1$-dimensional)

$$dS_t = b(t, S_t)dt + a(t, S_t)dW_t\,,\ \ t \in [0, T],$$
$$S_0 = s_0$$

S^0 ... riskless asset $S^1, \ldots, S^k$ risky assets

Special case: Black-Scholes model:

- Bond: $S_t^0 = S_0^0 \exp(rt)$
- Share: $S_t^1 = S_0^1 \exp\left(\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W_t\right),\ t \in [0, T],$

BS and SDE models

Consider SDE-model ($m+1$-dimensional)

$$dS_t = b(t, S_t)dt + a(t, S_t)dW_t\,,\ t \in [0, T],$$
$$S_0 = s_0$$

S^0 ... riskless asset $S^1, \ldots, S^k$ risky assets

Special case: Black-Scholes model:

- Bond: $S_t^0 = S_0^0 \exp(rt)$
- Share: $S_t^1 = S_0^1 \exp\left((\mu - \frac{\sigma^2}{2})t + \sigma W_t\right),\ t \in [0, T],$

Popular example: "Heston model"

- S^0 ... bond
- S^1 ... share
- S^2 ... volatility

Pricing European claims

- A European contingent claim is a contract that pays its owner an amount of money that depends on the price processes up to T

Pricing European claims

- A European contingent claim is a contract that pays its owner an amount of money that depends on the price processes up to T
  E.g. Asian Call option on S^1 pays $\max\left(\frac{1}{T-T_0}\int_{T_0}^{T} S_t^1 dt - K, 0\right)$ at time T

Pricing European claims

- A European contingent claim is a contract that pays its owner an amount of money that depends on the price processes up to T
  E.g. Asian Call option on S^1 pays $\max\left(\frac{1}{T-T_0}\int_{T_0}^{T} S_t^1 dt - K, 0\right)$ at time T
- for our purpose: any (reasonable) function on the set of paths of the price process

Pricing European claims

- A European contingent claim is a contract that pays its owner an amount of money that depends on the price processes up to T
  E.g. Asian Call option on S^1 pays $\max\left(\frac{1}{T-T_0}\int_{T_0}^{T} S_t^1 dt - K, 0\right)$ at time T
- for our purpose: any (reasonable) function on the set of paths of the price process
- Value of a claim C with payoff ψ

$$\pi_0(C) = \mathbb{E}_Q\left(\frac{S_0^0}{S_T^0}\psi(S^1, \ldots, S^k)\right)$$

Pricing European claims

- A European contingent claim is a contract that pays its owner an amount of money that depends on the price processes up to T
E.g. Asian Call option on S^1 pays $\max\left(\frac{1}{T-T_0}\int_{T_0}^{T} S_t^1 dt - K, 0\right)$ at time T
- for our purpose: any (reasonable) function on the set of paths of the price process
- Value of a claim C with payoff ψ

$$\pi_0(C) = \mathbb{E}_Q\left(\frac{S_0^0}{S_T^0}\psi(S^1, \ldots, S^k)\right)$$

where Q is a pricing measure, S^0 the riskless asset.

Pricing European claims

- A European contingent claim is a contract that pays its owner an amount of money that depends on the price processes up to T
  E.g. Asian Call option on S^1 pays $\max\left(\frac{1}{T-T_0}\int_{T_0}^{T} S_t^1 dt - K, 0\right)$ at time T
- for our purpose: any (reasonable) function on the set of paths of the price process
- Value of a claim C with payoff ψ

$$\pi_0(C) = \mathbb{E}_Q\left(\frac{S_0^0}{S_T^0}\psi(S^1,\ldots,S^k)\right)$$

  where Q is a pricing measure, S^0 the riskless asset.

- Compute $\pi_0(C)$ by (quasi-) Monte Carlo

Prices as integrals

- Compute paths with some method

Prices as integrals

- Compute paths with some method (Euler Maruyama, Milstein, . . .)
- from increments of Brownian motion

Prices as integrals

- Compute paths with some method (Euler Maruyama, Milstein, . . .)
- from increments of Brownian motion
- i.e. from d-dimensional standard normal input

Prices as integrals

- Compute paths with some method (Euler Maruyama, Milstein, . . .)
- from increments of Brownian motion
- i.e. from d-dimensional standard normal input
- where d can be a rather large number

Prices as integrals

- Compute paths with some method (Euler Maruyama, Milstein, ...)
- from increments of Brownian motion
- i.e. from d-dimensional standard normal input
- where d can be a rather large number

That means:

$$\pi_0(C) = \int_{\mathbb{R}^d} \hat{\psi}(x)\phi(x)dx$$

where ϕ is d-dimensional standard normal density

Prices as integrals

- Compute paths with some method (Euler Maruyama, Milstein, ...)
- from increments of Brownian motion
- i.e. from d-dimensional standard normal input
- where d can be a rather large number

That means:

$$\pi_0(C) = \int_{\mathbb{R}^d} \hat{\psi}(x)\phi(x)dx$$

where ϕ is d-dimensional standard normal density and $\hat{\psi}$ is a suitable reformulation of payoff ψ

Prices as integrals

- Compute paths with some method (Euler Maruyama, Milstein, . . .)
- from increments of Brownian motion
- i.e. from d-dimensional standard normal input
- where d can be a rather large number

That means:

$$\pi_0(C) = \int_{\mathbb{R}^d} \hat{\psi}(x)\phi(x)dx$$

where ϕ is d-dimensional standard normal density and $\hat{\psi}$ is a suitable reformulation of payoff ψ

Frequently this integral is transformed into one on the d-dimensional unitcube

Prices as integrals

- Compute paths with some method (Euler Maruyama, Milstein, . . .)
- from increments of Brownian motion
- i.e. from d-dimensional standard normal input
- where d can be a rather large number

That means:

$$\pi_0(C) = \int_{\mathbb{R}^d} \hat{\psi}(x)\phi(x)dx$$

where ϕ is d-dimensional standard normal density and $\hat{\psi}$ is a suitable reformulation of payoff ψ

Frequently this integral is transformed into one on the d-dimensional unitcube (but we won't)

High-dimensional integration

Suppose $f : (0,1)^d \longrightarrow \mathbb{R}$ is integrable and we want to know

$$I = \int_{(0,1)^d} f(\mathbf{x}) d\mathbf{x} \,.$$

High-dimensional integration

Suppose $f : (0,1)^d \longrightarrow \mathbb{R}$ is integrable and we want to know

$$I = \int_{(0,1)^d} f(\mathbf{x}) d\mathbf{x} \,.$$

Approximate

$$I \approx \frac{1}{N} \sum_{k=1}^{N} f(\mathbf{x}_k) \,,$$

High-dimensional integration

Suppose $f : (0,1)^d \longrightarrow \mathbb{R}$ is integrable and we want to know

$$I = \int_{(0,1)^d} f(\mathbf{x}) d\mathbf{x} \,.$$

Approximate

$$I \approx \frac{1}{N} \sum_{k=1}^{N} f(\mathbf{x}_k) \,,$$

- Monte Carlo: uniformly random points $\mathbf{x}_1, \ldots, \mathbf{x}_N \in (0,1)^d$

High-dimensional integration

Suppose $f : (0,1)^d \longrightarrow \mathbb{R}$ is integrable and we want to know

$$I = \int_{(0,1)^d} f(\mathbf{x}) d\mathbf{x} \,.$$

Approximate

$$I \approx \frac{1}{N} \sum_{k=1}^{N} f(\mathbf{x}_k) \,,$$

- Monte Carlo: uniformly random points $\mathbf{x}_1, \ldots, \mathbf{x}_N \in (0,1)^d$
- Quasi-Monte Carlo: well-distributed points $\mathbf{x}_1, \ldots, \mathbf{x}_N \in (0,1)^d$

High-dimensional integration, II

Typical error-estimate for QMC

$$\left| \int_{(0,1)^d} f(\mathbf{x}) d\mathbf{x} - \frac{1}{N} \sum_{k=1}^{N} f(\mathbf{x}_k) \right| \leq \|f\| \, D\left((\mathbf{x}_k)_{k=1}^{N} \right)$$

High-dimensional integration, II

Typical error-estimate for QMC

$$\left| \int_{(0,1)^d} f(\mathbf{x}) d\mathbf{x} - \frac{1}{N} \sum_{k=1}^{N} f(\mathbf{x}_k) \right| \leq \|f\| \, D\left((\mathbf{x}_k)_{k=1}^{N} \right)$$

where

- $\|.\|$ is some norm (or semi-) on a function space

High-dimensional integration, II

Typical error-estimate for QMC

$$\left| \int_{(0,1)^d} f(\mathbf{x}) d\mathbf{x} - \frac{1}{N} \sum_{k=1}^{N} f(\mathbf{x}_k) \right| \leq \|f\| \, D\left((\mathbf{x}_k)_{k=1}^{N} \right)$$

where

- $\|.\|$ is some norm (or semi-) on a function space measuring the variability of a function

High-dimensional integration, II

Typical error-estimate for QMC

$$\left| \int_{(0,1)^d} f(\mathbf{x}) d\mathbf{x} - \frac{1}{N} \sum_{k=1}^{N} f(\mathbf{x}_k) \right| \leq \|f\| \, D\left((\mathbf{x}_k)_{k=1}^{N} \right)$$

where

- $\|.\|$ is some norm (or semi-) on a function space measuring the variability of a function e.g. total variation in the sense of Hardy & Krause

High-dimensional integration, II

Typical error-estimate for QMC

$$\left| \int_{(0,1)^d} f(\mathbf{x}) d\mathbf{x} - \frac{1}{N} \sum_{k=1}^{N} f(\mathbf{x}_k) \right| \leq \|f\| \, D\left((\mathbf{x}_k)_{k=1}^{N} \right)$$

where

- $\|.\|$ is some norm (or semi-) on a function space measuring the variability of a function e.g. total variation in the sense of Hardy & Krause
- D is some measure of equi-distribution of N points in the unit cube

High-dimensional integration, II

Typical error-estimate for QMC

$$\left|\int_{(0,1)^d} f(\mathbf{x})d\mathbf{x} - \frac{1}{N}\sum_{k=1}^{N} f(\mathbf{x}_k)\right| \leq \|f\|\, D\left((\mathbf{x}_k)_{k=1}^{N}\right)$$

where

- $\|.\|$ is some norm (or semi-) on a function space measuring the variability of a function e.g. total variation in the sense of Hardy & Krause
- D is some measure of equi-distribution of N points in the unit cube, e.g. the star discrepancy D^*

High-dimensional integration, II

Typical error-estimate for QMC

$$\left| \int_{(0,1)^d} f(\mathbf{x}) d\mathbf{x} - \frac{1}{N} \sum_{k=1}^{N} f(\mathbf{x}_k) \right| \leq \|f\| \, D \left((\mathbf{x}_k)_{k=1}^{N} \right)$$

where

- $\|.\|$ is some norm (or semi-) on a function space measuring the variability of a function e.g. total variation in the sense of Hardy & Krause
- D is some measure of equi-distribution of N points in the unit cube, e.g. the star discrepancy D^*

"Koksma-Hlawka type error bound"

High-dimensional integration, III

Without going into details . . .

High-dimensional integration, III

Without going into details . . . the best constructions for uniformly distributed points give

$$D_N^* \leq C \frac{\log(N)^{d-1}}{N}$$

High-dimensional integration, III

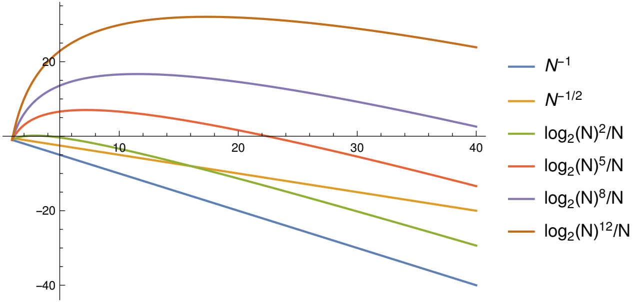
Without going into details . . . the best constructions for uniformly distributed points give

$$D_N^* \leq C\frac{\log(N)^{d-1}}{N}$$

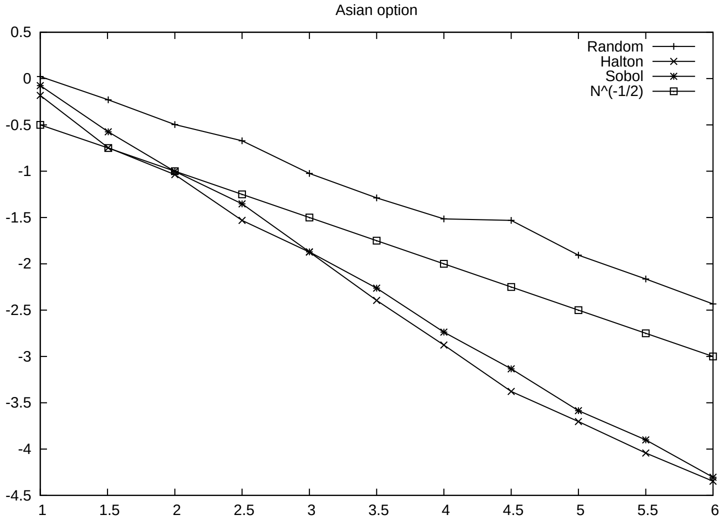
(For large N this convergence would be much faster than $N^{-\frac{1}{2}}$.)

High-dimensional integration, IV

Double logarithmic plot of $N \mapsto \frac{\log(N)^{d-1}}{N}$:



High-dimensional integration, V



Asian Option $d = 128$

High-dimensional integration, VI

This phenomenon frequently occured in applications from mathematical finance, or, more concretely, in derivative pricing.

High-dimensional integration, VI

This phenomenon frequently occured in applications from mathematical finance, or, more concretely, in derivative pricing.

Where does this apparent superiority come from?

Classical constructions

Three classical constructions of discrete Brownian paths from standard normal input $Z_1, \ldots, Z_d$:

Classical constructions

Three classical constructions of discrete Brownian paths from standard normal input $Z_1, \ldots, Z_d$:

- the forward method, a.k.a. step-by-step method or piecewise method

Classical constructions

Three classical constructions of discrete Brownian paths from standard normal input $Z_1, \dots, Z_d$:

- the forward method, a.k.a. step-by-step method or piecewise method $B_{\frac{k+1}{d}}$ is computed by adding $\sqrt{\frac{1}{d}} Z_{k+1}$ to $B_{\frac{k}{d}}$

Classical constructions

Three classical constructions of discrete Brownian paths from standard normal input $Z_1, \ldots, Z_d$:

- the forward method, a.k.a. step-by-step method or piecewise method $B_{\frac{k+1}{d}}$ is computed by adding $\sqrt{\frac{1}{d}} Z_{k+1}$ to $B_{\frac{k}{d}}$
- the Brownian bridge construction or Lévy-Ciesielski construction

Classical constructions

Three classical constructions of discrete Brownian paths from standard normal input $Z_1, \ldots, Z_d$:

- the forward method, a.k.a. step-by-step method or piecewise method
  $B_{\frac{k+1}{d}}$ is computed by adding $\sqrt{\frac{1}{d}} Z_{k+1}$ to $B_{\frac{k}{d}}$
- the Brownian bridge construction or Lévy-Ciesielski construction
  Compute first B_1 using Z_1, then $B_{1/2}$ using Z_2 and B_1,

Classical constructions

Three classical constructions of discrete Brownian paths from standard normal input $Z_1, \ldots, Z_d$:

- the forward method, a.k.a. step-by-step method or piecewise method
  $B_{\frac{k+1}{d}}$ is computed by adding $\sqrt{\frac{1}{d}} Z_{k+1}$ to $B_{\frac{k}{d}}$
- the Brownian bridge construction or Lévy-Ciesielski construction
  Compute first B_1 using Z_1, then $B_{1/2}$ using Z_2 and B_1, then $B_{1/4}$,

Classical constructions

Three classical constructions of discrete Brownian paths from standard normal input $Z_1, \ldots, Z_d$:

- the forward method, a.k.a. step-by-step method or piecewise method $B_{\frac{k+1}{d}}$ is computed by adding $\sqrt{\frac{1}{d}} Z_{k+1}$ to $B_{\frac{k}{d}}$
- the Brownian bridge construction or Lévy-Ciesielski construction Compute first B_1 using Z_1, then $B_{1/2}$ using Z_2 and B_1, then $B_{1/4}$, $B_{3/4}$, …, using already constructed neighbors

Classical constructions

Three classical constructions of discrete Brownian paths from standard normal input $Z_1, \ldots, Z_d$:

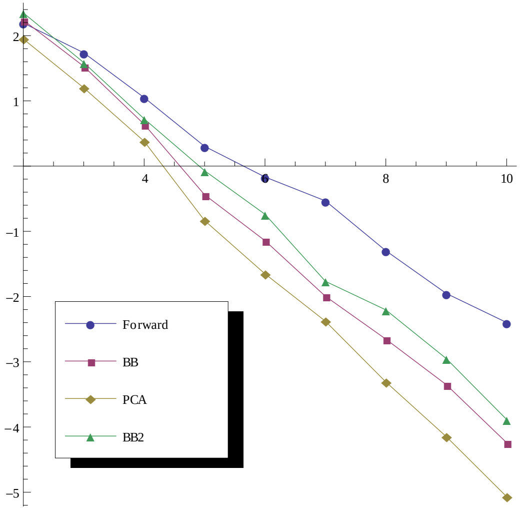
- the forward method, a.k.a. step-by-step method or piecewise method
  $B_{\frac{k+1}{d}}$ is computed by adding $\sqrt{\frac{1}{d}} Z_{k+1}$ to $B_{\frac{k}{d}}$
- the Brownian bridge construction or Lévy-Ciesielski construction
  Compute first B_1 using Z_1, then $B_{1/2}$ using Z_2 and B_1, then $B_{1/4}$, $B_{3/4}$, …, using already constructed neighbors
- the principal component analysis construction (PCA construction)

Classical constructions

Three classical constructions of discrete Brownian paths from standard normal input $Z_1, \ldots, Z_d$:

- the forward method, a.k.a. step-by-step method or piecewise method
  $B_{\frac{k+1}{d}}$ is computed by adding $\sqrt{\frac{1}{d}} Z_{k+1}$ to $B_{\frac{k}{d}}$
- the Brownian bridge construction or Lévy-Ciesielski construction
  Compute first B_1 using Z_1, then $B_{1/2}$ using Z_2 and B_1, then $B_{1/4}$, $B_{3/4}$, ..., using already constructed neighbors
- the principal component analysis construction (PCA construction)
  optimal ℓ^2 approximation of paths

Why we need more than one construction



Asian option in Heston model

Why we need more than one construction, II

Can we explain this behavior?

Why we need more than one construction, II

Can we explain this behavior?

- QMC seems to perform better if some of the variables are more important than the others

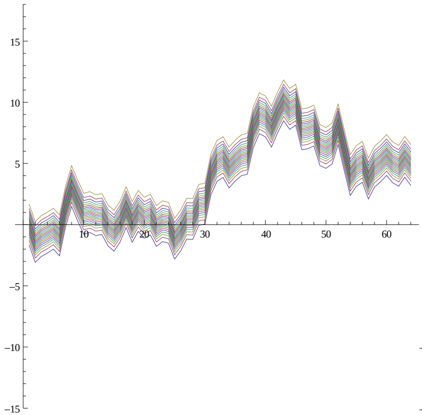
Why we need more than one construction, II

Can we explain this behavior?

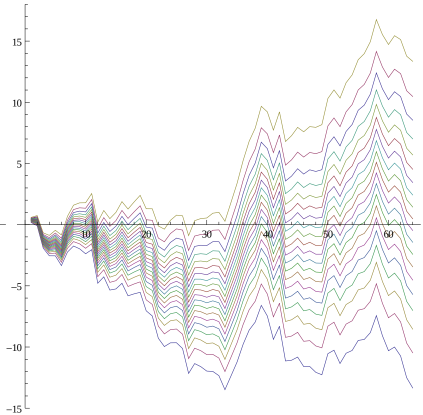
- QMC seems to perform better if some of the variables are more important than the others
- alternative path constructions often help to put more weight on the first few of the variables $Z_1, Z_2, \ldots, Z_d$

Why we need more than one construction, III

All variables but the first left constant:



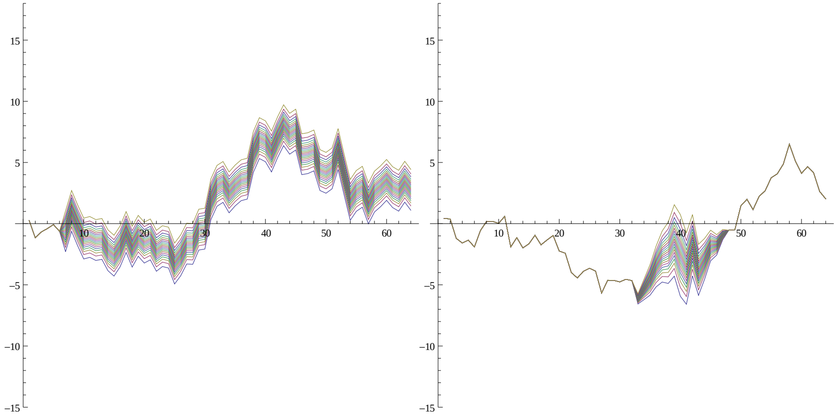
forward



brownian bridge

Why we need more than one construction, IV

All variables but the seventh left constant:



forward brownian bridge

Introduction of weights

Idea

- Consider weighted Korobov- or Sobolev spaces

Introduction of weights

Idea

- Consider weighted Korobov- or Sobolev spaces (Sloan and Woźniakowski, 1998)

Introduction of weights

Idea

- Consider weighted Korobov- or Sobolev spaces (Sloan and Woźniakowski, 1998)
- give Koksma-Hlawka type inequalities with weighted norm/discrepancy

Introduction of weights

Idea

- Consider weighted Korobov- or Sobolev spaces (Sloan and Woźniakowski, 1998)
- give Koksma-Hlawka type inequalities with weighted norm/discrepancy
- sequence need not be as well-distributed in coordinates that are less important

Orthogonal transforms

Papageorgiou (2002) oberved, that the classical constructions correspond to orthogonal transforms of the standard Gaussian input to forward construction

Orthogonal transforms

Papageorgiou (2002) oberved, that the classical constructions correspond to orthogonal transforms of the standard Gaussian input to forward construction

- Forward construction corresponds to identity

Orthogonal transforms

Papageorgiou (2002) oberved, that the classical constructions correspond to orthogonal transforms of the standard Gaussian input to forward construction

- Forward construction corresponds to identity
- Brownian bridge corresponds to inverse Haar transform

Orthogonal transforms

Papageorgiou (2002) oberved, that the classical constructions correspond to orthogonal transforms of the standard Gaussian input to forward construction

- Forward construction corresponds to identity
- Brownian bridge corresponds to inverse Haar transform
- PCA corresponds to (fast) sine transform

Orthogonal transforms

Papageorgiou (2002) oberved, that the classical constructions correspond to orthogonal transforms of the standard Gaussian input to forward construction

- Forward construction corresponds to identity
- Brownian bridge corresponds to inverse Haar transform
- PCA corresponds to (fast) sine transform

L.(2012) provides a number of alternative constructions

Orthogonal transforms

Papageorgiou (2002) oberved, that the classical constructions correspond to orthogonal transforms of the standard Gaussian input to forward construction

- Forward construction corresponds to identity
- Brownian bridge corresponds to inverse Haar transform
- PCA corresponds to (fast) sine transform

L.(2012) provides a number of alternative constructions with cost proportional to $\log(d)d$ per path

Orthogonal transforms

Papageorgiou (2002) oberved, that the classical constructions correspond to orthogonal transforms of the standard Gaussian input to forward construction

- Forward construction corresponds to identity
- Brownian bridge corresponds to inverse Haar transform
- PCA corresponds to (fast) sine transform

L.(2012) provides a number of alternative constructions with cost proportional to $\log(d)d$ per path

However:

Orthogonal transforms

Papageorgiou (2002) oberved, that the classical constructions correspond to orthogonal transforms of the standard Gaussian input to forward construction

- Forward construction corresponds to identity
- Brownian bridge corresponds to inverse Haar transform
- PCA corresponds to (fast) sine transform

L.(2012) provides a number of alternative constructions with cost proportional to $\log(d)d$ per path

However: Whether a path construction is "good" or not depends on the payoff as well

Hermite space on $\mathbb{R}$

- $\phi(x) := \frac{1}{\sqrt{2\pi}} \exp(-\frac{x^2}{2})$, $x \in \mathbb{R}$

Hermite space on $\mathbb{R}$

- $\phi(x) := \frac{1}{\sqrt{2\pi}} \exp(-\frac{x^2}{2})$, $x \in \mathbb{R}$
- $L^2(\mathbb{R}, \phi) = \{f$: measurable and $\int_{\mathbb{R}} |f|^2 \phi < \infty\}$

Hermite space on $\mathbb{R}$

- $\phi(x) := \frac{1}{\sqrt{2\pi}} \exp(-\frac{x^2}{2})$, $x \in \mathbb{R}$
- $L^2(\mathbb{R}, \phi) = \{f$: measurable and $\int_{\mathbb{R}} |f|^2 \phi < \infty\}$
- $(\bar{H}_k)_k$ ... sequence of normalized Hermite polynomials

Hermite space on $\mathbb{R}$

- $\phi(x) := \frac{1}{\sqrt{2\pi}} \exp(-\frac{x^2}{2})$, $x \in \mathbb{R}$
- $L^2(\mathbb{R}, \phi) = \{f$: measurable and $\int_{\mathbb{R}} |f|^2 \phi < \infty\}$
- $(\bar{H}_k)_k$ ... sequence of normalized Hermite polynomials (i.e. $\bar{H}_0, \bar{H}_1, \bar{H}_2, \ldots$ is the Gram-Schmidt orthogonalization of $1, x, x^2, \ldots$ in $L^2(\mathbb{R}, \phi)$

Hermite space on $\mathbb{R}$

- $\phi(x) := \frac{1}{\sqrt{2\pi}} \exp(-\frac{x^2}{2})$, $x \in \mathbb{R}$
- $L^2(\mathbb{R}, \phi) = \{f : \text{measurable and } \int_{\mathbb{R}} |f|^2 \phi < \infty\}$
- $(\bar{H}_k)_k$ ... sequence of normalized Hermite polynomials (i.e. $\bar{H}_0, \bar{H}_1, \bar{H}_2, \ldots$ is the Gram-Schmidt orthogonalization of $1, x, x^2, \ldots$ in $L^2(\mathbb{R}, \phi)$)
- $(\bar{H}_k)_k$ ... forms Hilbert space basis of $L^2(\mathbb{R}, \phi)$,

Hermite space on $\mathbb{R}$

- $\phi(x) := \frac{1}{\sqrt{2\pi}} \exp(-\frac{x^2}{2})$, $x \in \mathbb{R}$
- $L^2(\mathbb{R}, \phi) = \{f : \text{measurable and } \int_{\mathbb{R}} |f|^2 \phi < \infty\}$
- $(\bar{H}_k)_k$ ... sequence of normalized Hermite polynomials (i.e. $\bar{H}_0, \bar{H}_1, \bar{H}_2, \ldots$ is the Gram-Schmidt orthogonalization of $1, x, x^2, \ldots$ in $L^2(\mathbb{R}, \phi)$)
- $(\bar{H}_k)_k$ ... forms Hilbert space basis of $L^2(\mathbb{R}, \phi)$, i.e.

$$f = \sum_{k \geq 0} \hat{f}(k) \bar{H}_k \qquad \text{in } L^2(\mathbb{R}, \phi)$$

Hermite space on $\mathbb{R}$

- $\phi(x) := \frac{1}{\sqrt{2\pi}} \exp(-\frac{x^2}{2})$, $x \in \mathbb{R}$
- $L^2(\mathbb{R}, \phi) = \{f : \text{measurable and } \int_{\mathbb{R}} |f|^2 \phi < \infty\}$
- $(\bar{H}_k)_k$ ... sequence of normalized Hermite polynomials (i.e. $\bar{H}_0, \bar{H}_1, \bar{H}_2, \ldots$ is the Gram-Schmidt orthogonalization of $1, x, x^2, \ldots$ in $L^2(\mathbb{R}, \phi)$
- $(\bar{H}_k)_k$ ... forms Hilbert space basis of $L^2(\mathbb{R}, \phi)$, i.e.

$$f = \sum_{k \geq 0} \hat{f}(k) \bar{H}_k \qquad \text{in } L^2(\mathbb{R}, \phi)$$

- $\hat{f}(k) = \int_{\mathbb{R}} f(x) \bar{H}_k(x) \phi(x) dx$

Hermite space on $\mathbb{R}$, II

Theorem (Irrgeher & L. (?) (2015))

Let $(r_k)_{k\geq 0}$ *be a sequence with*

- $r_k > 0$

Hermite space on $\mathbb{R}$, II

Theorem (Irrgeher & L. (?) (2015))

Let $(r_k)_{k\geq 0}$ *be a sequence with*

- $r_k > 0$
- $\sum_{k\geq 0} r_k < \infty$

Hermite space on $\mathbb{R}$, II

Theorem (Irrgeher & L. (?) (2015))

Let $(r_k)_{k\geq 0}$ *be a sequence with*

- $r_k > 0$
- $\sum_{k\geq 0} r_k < \infty$

If $f:\mathbb{R}\to\mathbb{R}$ *is continuous,* $\int_{\mathbb{R}} f(x)^2\phi(x)dx < \infty$*, and* $\sum_{k\geq 0} r_k^{-1}|\hat{f}(k)|^2 < \infty$

Hermite space on $\mathbb{R}$, II

Theorem (Irrgeher & L. (?) (2015))

Let $(r_k)_{k\geq 0}$ *be a sequence with*

- $r_k > 0$
- $\sum_{k\geq 0} r_k < \infty$

If $f : \mathbb{R} \to \mathbb{R}$ *is continuous,* $\int_{\mathbb{R}} f(x)^2 \phi(x) dx < \infty$*, and* $\sum_{k\geq 0} r_k^{-1} |\hat{f}(k)|^2 < \infty$ *then*

$$f(x) = \sum_{k\geq 0} \hat{f}(k) \bar{H}_k(x) \qquad \textit{for all } x \in \mathbb{R}$$

Hermite space on $\mathbb{R}$, III

- Fix some positive summable sequence $r = (r_k)_{k \geq 0}$

Hermite space on $\mathbb{R}$, III

- Fix some positive summable sequence $r = (r_k)_{k \geq 0}$
- Introdude the norm

$$\|f\|_{\mathrm{her}}^2 := \sum_{k=0}^{\infty} r_k^{-1} \hat{f}(k)^2$$

Hermite space on $\mathbb{R}$, III

- Fix some positive summable sequence $r = (r_k)_{k\geq 0}$
- Introduce the norm

$$\|f\|_{\mathrm{her}}^2 := \sum_{k=0}^{\infty} r_k^{-1} \hat{f}(k)^2$$

- and inner product:

$$\langle f, g\rangle_{\mathrm{her}} := \sum_{k=0}^{\infty} r_k^{-1} \hat{f}(k)\hat{g}(k)$$

Hermite space on $\mathbb{R}$, IV

Theorem (Irrgeher & L.(2015))

The Hilbert space

$$\mathscr{H}_{\mathrm{her}}(\mathbb{R}) := \{f \in L^2(\mathbb{R}, \phi) \cap C(\mathbb{R}) : \|f\|_{\mathrm{her}} < \infty\}$$

is a reproducing kernel Hilbert space with reproducing kernel

$$K_{\mathrm{her}}(x, y) = \sum_{k \in \mathbb{N}_0} r(k) \bar{H}_k(x) \bar{H}_k(y)$$

"one-dimensional" Hermite space

Hermite space on $\mathbb{R}$, V

- There are indeed some interesting functions in $\mathscr{H}_{\mathrm{her}}(\mathbb{R})$:

Hermite space on $\mathbb{R}$, V

- There are indeed some interesting functions in $\mathscr{H}_{\mathrm{her}}(\mathbb{R})$:
  Irrgeher & L.(2015): For $r_k = k^{-\alpha}$, $f \in \mathscr{H}_{\mathrm{her}}(\mathbb{R})$ contains all functions for which derivatives up to order $\beta > \alpha + 1$ exist and satisfy an integrability and growth condition

Hermite space on $\mathbb{R}$, V

- There are indeed some interesting functions in $\mathscr{H}_{\mathrm{her}}(\mathbb{R})$:
  Irrgeher & L.(2015): For $r_k = k^{-\alpha}$, $f \in \mathscr{H}_{\mathrm{her}}(\mathbb{R})$ contains all functions for which derivatives up to order $\beta > \alpha + 1$ exist and satisfy an integrability and growth condition
- Newer result: Dick, Irrgeher, L., Pillichshammer (2016):

Hermite space on $\mathbb{R}$, V

- There are indeed some interesting functions in $\mathscr{H}_{\text{her}}(\mathbb{R})$:
  Irrgeher & L.(2015): For $r_k = k^{-\alpha}$, $f \in \mathscr{H}_{\text{her}}(\mathbb{R})$ contains all functions for which derivatives up to order $\beta > \alpha + 1$ exist and satisfy an integrability and growth condition
- Newer result: Dick, Irrgeher, L., Pillichshammer (2016): For every $\alpha \geq 1$ there exists a (unique) sequence $(r_{\alpha,k})_{k\in\mathbb{N}_0}$

Hermite space on $\mathbb{R}$, V

- There are indeed some interesting functions in $\mathscr{H}_{\mathrm{her}}(\mathbb{R})$:
  Irrgeher & L.(2015): For $r_k = k^{-\alpha}$, $f \in \mathscr{H}_{\mathrm{her}}(\mathbb{R})$ contains all functions for which derivatives up to order $\beta > \alpha + 1$ exist and satisfy an integrability and growth condition
- Newer result: Dick, Irrgeher, L., Pillichshammer (2016): For every $\alpha \geq 1$ there exists a (unique) sequence $(r_{\alpha,k})_{k\in\mathbb{N}_0}$ with $\lim_{k\to\infty} r_{\alpha,k} k^{\alpha} = 1$

Hermite space on $\mathbb{R}$, V

- There are indeed some interesting functions in $\mathscr{H}_{\mathrm{her}}(\mathbb{R})$: Irrgeher & L.(2015): For $r_k = k^{-\alpha}$, $f \in \mathscr{H}_{\mathrm{her}}(\mathbb{R})$ contains all functions for which derivatives up to order $\beta > \alpha + 1$ exist and satisfy an integrability and growth condition
- Newer result: Dick, Irrgeher, L., Pillichshammer (2016): For every $\alpha \geq 1$ there exists a (unique) sequence $(r_{\alpha,k})_{k\in\mathbb{N}_0}$ with $\lim_{k\to\infty} r_{\alpha,k} k^{\alpha} = 1$ and

$$\|f\|_{\mathrm{her}}^2 = \sum_{k=0}^{\infty} r_{\alpha,k}^{-1} |\hat{f}(k)|^2 = \sum_{j=0}^{\alpha} \int_{\mathbb{R}} |f^{(j)}(x)|^2 \phi(x) dx$$

Hermite space on $\mathbb{R}$, V

- There are indeed some interesting functions in $\mathscr{H}_{\mathrm{her}}(\mathbb{R})$: Irrgeher & L.(2015): For $r_k = k^{-\alpha}$, $f \in \mathscr{H}_{\mathrm{her}}(\mathbb{R})$ contains all functions for which derivatives up to order $\beta > \alpha + 1$ exist and satisfy an integrability and growth condition
- Newer result: Dick, Irrgeher, L., Pillichshammer (2016): For every $\alpha \geq 1$ there exists a (unique) sequence $(r_{\alpha,k})_{k\in\mathbb{N}_0}$ with $\lim_{k\to\infty} r_{\alpha,k} k^{\alpha} = 1$ and

$$\|f\|_{\mathrm{her}}^2 = \sum_{k=0}^{\infty} r_{\alpha,k}^{-1} |\hat{f}(k)|^2 = \sum_{j=0}^{\alpha} \int_{\mathbb{R}} |f^{(j)}(x)|^2 \phi(x) dx$$

- That is, for this sequence the Hermite-space is isometrically isomorphic to a certain classical Sobolev space

Hermite spaces on $\mathbb{R}^d$

- For a d-multi-index $\mathbf{k} = (k_1, \ldots, k_d)$ define

Hermite spaces on $\mathbb{R}^d$

- For a d-multi-index $\mathbf{k} = (k_1, \ldots, k_d)$ define

$$\bar{H}_{\mathbf{k}}(x_1, \ldots, x_d) := \prod_{j=1}^{d} \bar{H}_{k_j}(x_j)$$

Hermite spaces on $\mathbb{R}^d$

- For a d-multi-index $\mathbf{k} = (k_1, \ldots, k_d)$ define

$$\bar{H}_{\mathbf{k}}(x_1, \ldots, x_d) := \prod_{j=1}^{d} \bar{H}_{k_j}(x_j)$$

- defines Hilbert space basis of $L^2(\mathbb{R}^d, \phi)$

Hermite spaces on $\mathbb{R}^d$

- For a d-multi-index $\mathbf{k} = (k_1, \ldots, k_d)$ define

$$\bar{H}_{\mathbf{k}}(x_1, \ldots, x_d) := \prod_{j=1}^{d} \bar{H}_{k_j}(x_j)$$

- defines Hilbert space basis of $L^2(\mathbb{R}^d, \phi)$

$$\phi(\mathbf{x}) := \prod_{j=1}^{d} \phi(x_j)$$

Hermite spaces on $\mathbb{R}^d$

- For a d-multi-index $\mathbf{k} = (k_1, \ldots, k_d)$ define

$$\bar{H}_{\mathbf{k}}(x_1, \ldots, x_d) := \prod_{j=1}^{d} \bar{H}_{k_j}(x_j)$$

- defines Hilbert space basis of $L^2(\mathbb{R}^d, \phi)$

$$\phi(\mathbf{x}) := \prod_{j=1}^{d} \phi(x_j)$$

- write $\hat{f}(\mathbf{k}) := \langle f, \bar{H}_{\mathbf{k}} \rangle = \int_{\mathbb{R}^d} f(\mathbf{x}) \bar{H}_{\mathbf{k}}(\mathbf{x}) \phi(\mathbf{x}) d\mathbf{x}$

Hermite spaces on $\mathbb{R}^d$, II

Fix a positive summable sequence $(r_k)_{k\in\mathbb{N}_0}$

Hermite spaces on $\mathbb{R}^d$, II

Fix a positive summable sequence $(r_k)_{k\in\mathbb{N}_0}$

- For given coordinate weights $\gamma_1 \geq \gamma_2 \geq \cdots > 0$ let the function $\mathbf{r}(\gamma, .) : \mathbb{N}_0^d \longrightarrow \mathbb{R}$ be given by

$$\mathbf{r}(\gamma, \mathbf{k}) = \prod_{j=1}^{d} \tilde{r}(\gamma_j, k_j)$$

Hermite spaces on $\mathbb{R}^d$, II

Fix a positive summable sequence $(r_k)_{k\in\mathbb{N}_0}$

- For given coordinate weights $\gamma_1 \geq \gamma_2 \geq \cdots > 0$ let the function $\mathbf{r}(\gamma, .) : \mathbb{N}_0^d \longrightarrow \mathbb{R}$ be given by

$$\mathbf{r}(\gamma, \mathbf{k}) = \prod_{j=1}^{d} \tilde{r}(\gamma_j, k_j)$$

where

$$\tilde{r}(\gamma, k) := \begin{cases} 1 & k = 0 \\ \gamma^{-1} r_k & k \geq 1 \end{cases}$$

Hermite spaces on $\mathbb{R}^d$, II

Fix a positive summable sequence $(r_k)_{k\in\mathbb{N}_0}$

- For given coordinate weights $\gamma_1 \geq \gamma_2 \geq \cdots > 0$ let the function $\mathbf{r}(\gamma, .) : \mathbb{N}_0^d \longrightarrow \mathbb{R}$ be given by

$$\mathbf{r}(\boldsymbol{\gamma}, \mathbf{k}) = \prod_{j=1}^{d} \tilde{r}(\gamma_j, k_j)$$

where

$$\tilde{r}(\gamma, k) := \begin{cases} 1 & k = 0 \\ \gamma^{-1} r_k & k \geq 1 \end{cases}$$

- And consider the inner product

$$\langle f, g \rangle_{\mathrm{her},\boldsymbol{\gamma}} = \sum_{\mathbf{k}\in\mathbb{N}_0^d} \mathbf{r}(\boldsymbol{\gamma}, \mathbf{k})^{-1} \hat{f}(\mathbf{k}) \hat{g}(\mathbf{k})$$

Hermite spaces on $\mathbb{R}^d$, II

Fix a positive summable sequence $(r_k)_{k\in\mathbb{N}_0}$

- For given coordinate weights $\gamma_1 \geq \gamma_2 \geq \cdots > 0$ let the function $\mathbf{r}(\gamma, .) : \mathbb{N}_0^d \longrightarrow \mathbb{R}$ be given by

$$\mathbf{r}(\boldsymbol{\gamma}, \mathbf{k}) = \prod_{j=1}^{d} \tilde{r}(\gamma_j, k_j)$$

  where

$$\tilde{r}(\gamma, k) := \begin{cases} 1 & k = 0 \\ \gamma^{-1} r_k & k \geq 1 \end{cases}$$

- And consider the inner product

$$\langle f, g \rangle_{\mathrm{her},\boldsymbol{\gamma}} = \sum_{\mathbf{k}\in\mathbb{N}_0^d} \mathbf{r}(\boldsymbol{\gamma}, \mathbf{k})^{-1} \hat{f}(\mathbf{k}) \hat{g}(\mathbf{k})$$

  Let $\mathscr{H}_{\mathrm{her},\boldsymbol{\gamma}}(\mathbb{R}^d)$ be the corresponding Hilbert space

Hermite spaces on $\mathbb{R}^d$, III

- Irrgeher & L.(2015): Integration in the RKHS $\mathscr{H}_{\mathrm{her},\boldsymbol{\gamma}}(\mathbb{R}^d)$ is

Hermite spaces on $\mathbb{R}^d$, III

- Irrgeher & L.(2015): Integration in the RKHS $\mathscr{H}_{\mathrm{her},\boldsymbol{\gamma}}(\mathbb{R}^d)$ is
  - strongly tractable if $\sum_{j=1}^{\infty} \gamma_j < \infty$,

Hermite spaces on $\mathbb{R}^d$, III

- Irrgeher & L.(2015): Integration in the RKHS $\mathscr{H}_{\mathrm{her},\boldsymbol{\gamma}}(\mathbb{R}^d)$ is
  - strongly tractable if $\sum_{j=1}^{\infty} \gamma_j < \infty$,
  - tractable if $\limsup_d \frac{1}{\log d} \sum_{j=1}^{d} \gamma_j < \infty$.

Hermite spaces on $\mathbb{R}^d$, III

- Irrgeher & L.(2015): Integration in the RKHS $\mathscr{H}_{\mathrm{her},\gamma}(\mathbb{R}^d)$ is
  - strongly tractable if $\sum_{j=1}^{\infty} \gamma_j < \infty$,
  - tractable if $\limsup_d \frac{1}{\log d} \sum_{j=1}^{d} \gamma_j < \infty$.
- I.e., integration does not necessarily suffer from curse of dimension

Hermite spaces on $\mathbb{R}^d$, III

- Irrgeher & L.(2015): Integration in the RKHS $\mathscr{H}_{\mathrm{her},\boldsymbol{\gamma}}(\mathbb{R}^d)$ is
  - strongly tractable if $\sum_{j=1}^{\infty} \gamma_j < \infty$,
  - tractable if $\limsup_d \frac{1}{\log d} \sum_{j=1}^{d} \gamma_j < \infty$.
- I.e., integration does not necessarily suffer from curse of dimension
- Why are we interested in this particular space?

Hermite spaces on $\mathbb{R}^d$, III

- Irrgeher & L.(2015): Integration in the RKHS $\mathscr{H}_{\mathrm{her},\boldsymbol{\gamma}}(\mathbb{R}^d)$ is
  - strongly tractable if $\sum_{j=1}^{\infty} \gamma_j < \infty$,
  - tractable if $\limsup_d \frac{1}{\log d} \sum_{j=1}^{d} \gamma_j < \infty$.
- I.e., integration does not necessarily suffer from curse of dimension
- Why are we interested in this particular space?
  - Let $f \in \mathscr{H}_{\mathrm{her},\boldsymbol{\gamma}}$ and let $U : \mathbb{R}^d \longrightarrow \mathbb{R}^d$ some orthogonal transform, $U^\top U = 1_{\mathbb{R}^d}$

Hermite spaces on $\mathbb{R}^d$, III

- Irrgeher & L.(2015): Integration in the RKHS $\mathscr{H}_{\mathrm{her},\boldsymbol{\gamma}}(\mathbb{R}^d)$ is
  - strongly tractable if $\sum_{j=1}^{\infty} \gamma_j < \infty$,
  - tractable if $\limsup_d \frac{1}{\log d} \sum_{j=1}^{d} \gamma_j < \infty$.
- I.e., integration does not necessarily suffer from curse of dimension
- Why are we interested in this particular space?
  - Let $f \in \mathscr{H}_{\mathrm{her},\boldsymbol{\gamma}}$ and let $U : \mathbb{R}^d \longrightarrow \mathbb{R}^d$ some orthogonal transform, $U^\top U = 1_{\mathbb{R}^d}$
  - then $f \circ U \in \mathscr{H}_{\mathrm{her},\boldsymbol{\gamma}}$

Hermite spaces on $\mathbb{R}^d$, III

- Irrgeher & L.(2015): Integration in the RKHS $\mathscr{H}_{\mathrm{her},\boldsymbol{\gamma}}(\mathbb{R}^d)$ is
  - strongly tractable if $\sum_{j=1}^{\infty} \gamma_j < \infty$,
  - tractable if $\limsup_d \frac{1}{\log d} \sum_{j=1}^{d} \gamma_j < \infty$.
- I.e., integration does not necessarily suffer from curse of dimension
- Why are we interested in this particular space?
  - Let $f \in \mathscr{H}_{\mathrm{her},\boldsymbol{\gamma}}$ and let $U : \mathbb{R}^d \longrightarrow \mathbb{R}^d$ some orthogonal transform, $U^\top U = 1_{\mathbb{R}^d}$
  - then $f \circ U \in \mathscr{H}_{\mathrm{her},\boldsymbol{\gamma}}$
  - also $\int_{\mathbb{R}^d} f \circ U(\mathbf{x})\phi(\mathbf{x})d\mathbf{x} = \int_{\mathbb{R}^d} f(\mathbf{x})\phi(\mathbf{x})d\mathbf{x}$

Hermite spaces on $\mathbb{R}^d$, III

- Irrgeher & L.(2015): Integration in the RKHS $\mathscr{H}_{\mathrm{her},\boldsymbol{\gamma}}(\mathbb{R}^d)$ is
  - strongly tractable if $\sum_{j=1}^{\infty} \gamma_j < \infty$,
  - tractable if $\limsup_d \frac{1}{\log d} \sum_{j=1}^{d} \gamma_j < \infty$.
- I.e., integration does not necessarily suffer from curse of dimension
- Why are we interested in this particular space?
  - Let $f \in \mathscr{H}_{\mathrm{her},\boldsymbol{\gamma}}$ and let $U : \mathbb{R}^d \longrightarrow \mathbb{R}^d$ some orthogonal transform, $U^\top U = 1_{\mathbb{R}^d}$
  - then $f \circ U \in \mathscr{H}_{\mathrm{her},\boldsymbol{\gamma}}$
  - also $\int_{\mathbb{R}^d} f \circ U(\mathbf{x})\phi(\mathbf{x})d\mathbf{x} = \int_{\mathbb{R}^d} f(\mathbf{x})\phi(\mathbf{x})d\mathbf{x}$
  - but in general $\|f \circ U\|_{\mathrm{her},\boldsymbol{\gamma}} \neq \|f\|_{\mathrm{her},\boldsymbol{\gamma}}$

Hermite spaces on $\mathbb{R}^d$, III

- Irrgeher & L.(2015): Integration in the RKHS $\mathscr{H}_{\mathrm{her},\boldsymbol{\gamma}}(\mathbb{R}^d)$ is
  - strongly tractable if $\sum_{j=1}^{\infty} \gamma_j < \infty$,
  - tractable if $\limsup_d \frac{1}{\log d} \sum_{j=1}^{d} \gamma_j < \infty$.
- I.e., integration does not necessarily suffer from curse of dimension
- Why are we interested in this particular space?
  - Let $f \in \mathscr{H}_{\mathrm{her},\boldsymbol{\gamma}}$ and let $U : \mathbb{R}^d \longrightarrow \mathbb{R}^d$ some orthogonal transform, $U^\top U = 1_{\mathbb{R}^d}$
  - then $f \circ U \in \mathscr{H}_{\mathrm{her},\boldsymbol{\gamma}}$
  - also $\int_{\mathbb{R}^d} f \circ U(\mathbf{x})\phi(\mathbf{x})d\mathbf{x} = \int_{\mathbb{R}^d} f(\mathbf{x})\phi(\mathbf{x})d\mathbf{x}$
  - but in general $\|f \circ U\|_{\mathrm{her},\boldsymbol{\gamma}} \neq \|f\|_{\mathrm{her},\boldsymbol{\gamma}}$
  - note difference to Monte Carlo

Hermite spaces on $\mathbb{R}^d$, IV

- norm of $\|f \circ U\|$ depends on U in a continuous fashion.

Hermite spaces on $\mathbb{R}^d$, IV

- norm of $\|f \circ U\|$ depends on U in a continuous fashion.
- We can – in principle – use optimization techniques to find best transform

Hermite spaces on $\mathbb{R}^d$, IV

- norm of $\|f \circ U\|$ depends on U in a continuous fashion.
- We can – in principle – use optimization techniques to find best transform
- An earlier result/method by Irrgeher & L. (2012) is better understood in the context of Hermite spaces

Hermite spaces on $\mathbb{R}^d$, IV

- norm of $\|f \circ U\|$ depends on U in a continuous fashion.
- We can – in principle – use optimization techniques to find best transform
- An earlier result/method by Irrgeher & L. (2012) is better understood in the context of Hermite spaces
- instead of minimizing the weighted norm of $\|f \circ U\|$, minimize a seminorm which does not take into account all Hermite coefficients

Hermite spaces on $\mathbb{R}^d$, IV

- norm of $\|f \circ U\|$ depends on U in a continuous fashion.
- We can – in principle – use optimization techniques to find best transform
- An earlier result/method by Irrgeher & L. (2012) is better understood in the context of Hermite spaces
- instead of minimizing the weighted norm of $\|f \circ U\|$, minimize a seminorm which does not take into account all Hermite coefficients
- for example, only consider order one coefficients

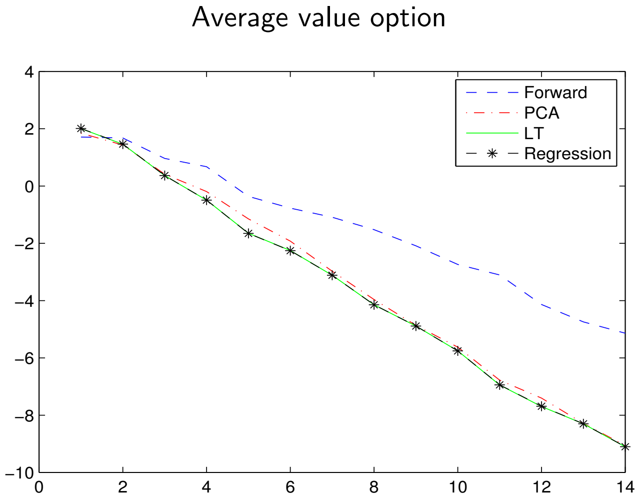
Hermite spaces on $\mathbb{R}^d$, IV

- norm of $\|f \circ U\|$ depends on U in a continuous fashion.
- We can – in principle – use optimization techniques to find best transform
- An earlier result/method by Irrgeher & L. (2012) is better understood in the context of Hermite spaces
- instead of minimizing the weighted norm of $\|f \circ U\|$, minimize a seminorm which does not take into account all Hermite coefficients
- for example, only consider order one coefficients
- method is termed linear regression method

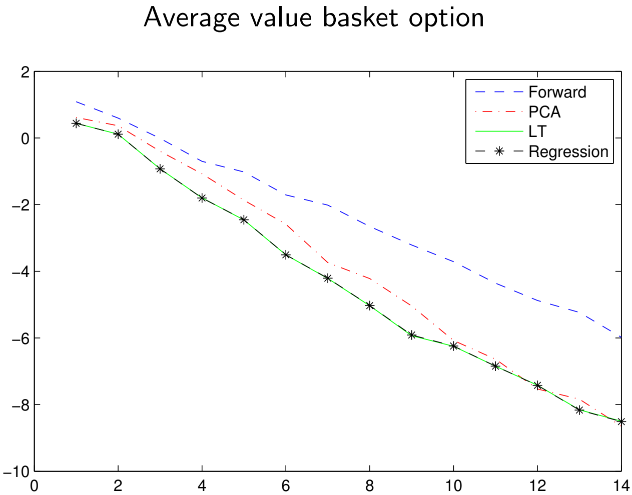
Hermite spaces on $\mathbb{R}^d$, IV

- norm of $\|f \circ U\|$ depends on U in a continuous fashion.
- We can – in principle – use optimization techniques to find best transform
- An earlier result/method by Irrgeher & L. (2012) is better understood in the context of Hermite spaces
- instead of minimizing the weighted norm of $\|f \circ U\|$, minimize a seminorm which does not take into account all Hermite coefficients
- for example, only consider order one coefficients
- method is termed linear regression method and generates paths in linear time

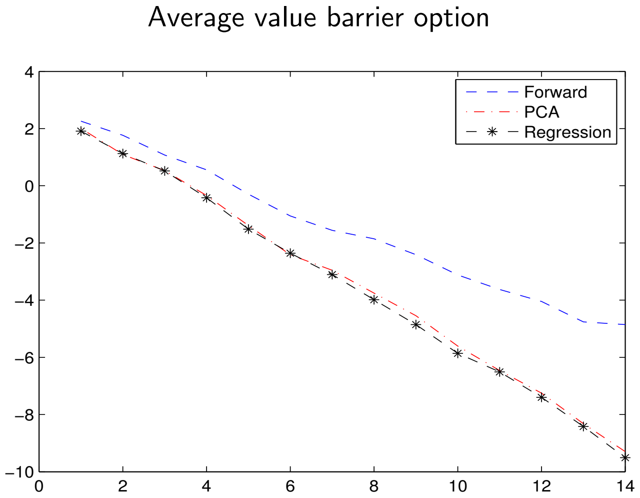
Examples regression algorithm



Examples regression algorithm, II



Examples regression algorithm, III



Hermite spaces

Conclusion

- We have provided a potential approach to explaining the effectiveness of QMC for high-dimensional financial applications

Hermite spaces

Conclusion

- We have provided a potential approach to explaining the effectiveness of QMC for high-dimensional financial applications
- the approach enabled us to find a method that is practically the best available at the moment

Hermite spaces
Conclusion

- We have provided a potential approach to explaining the effectiveness of QMC for high-dimensional financial applications
- the approach enabled us to find a method that is practically the best available at the moment
- different lines of research:
    - construct point sets/sequences for those spaces

Hermite spaces

Conclusion

- We have provided a potential approach to explaining the effectiveness of QMC for high-dimensional financial applications
- the approach enabled us to find a method that is practically the best available at the moment
- different lines of research:
  - construct point sets/sequences for those spaces
  - generalize regression method to higher oder approximations

Hermite spaces

Conclusion

- We have provided a potential approach to explaining the effectiveness of QMC for high-dimensional financial applications
- the approach enabled us to find a method that is practically the best available at the moment
- different lines of research:
  - construct point sets/sequences for those spaces
  - generalize regression method to higher oder approximations
  - deal with “kinks”

Thank you !

- C. Irrgeher, G. Leobacher: High-dimensional integration on $\mathbb{R}^d$, weighted Hermite spaces, and orthogonal transforms. J. Complexity (31), pp. 174-205. 2015
- C. Irrgeher, G. Leobacher: Fast orthogonal transforms for pricing derivatives with quasi-Monte Carlo, in: Proceedings of the Winter Simulation Conference 2012, 2012.
- G. Leobacher: Fast orthogonal transforms and generation of Brownian paths. Journal of Complexity (28), pp. 278-302. 2012
- I. Sloan, H. Woǹiakowski: When Are Quasi-Monte Carlo Algorithms Efficient for High Dimensional Integrals? Journal of Complexity (14), pp. 1–33, 1998.
- A. Papageorgiou: The Brownian Bridge Does Not Offer a Consistent Advantage in Quasi-Monte Carlo Integration. Journal of Complexity (18), pp. 171–186, 2002.