Hamiltonian Monte Carlo sampling for Wishart distributions with eigenvalue constraints

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Abstract

Sampling from constrained target spaces for Bayesian inference is a non-trivial problem. A recent development has been the use of Hamiltonian Monte Carlo in combination with particle reflection, see [4]. However, Hamiltonian Monte Carlo is sensitive to several hyper parameters, that need to be tuned, to ensure an efficient sampler. For this purpose, [5] suggested a black box algorithm that handles this problem. Our approach is to combine the two former ideas to solve the problem of sampling Wishart distributed matrices with eigenvalue constraints. Therefore, we exploit the eigenvalue decomposition of positive definite matrices. The suggested method performs better than the initial sampler of [3] when the dimension of the target space grows. Important applications of our sampler are the normal hierarchical model of [2] and the rank test in a principal component analysis as in [1].

References

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